Find the first six terms of each sequence.

1. \( a_n = 2n + 1 \)
2. \( a_n = 3n - 1 \)
3. \( a_n = 2^n + 1 \)
4. \( a_n = 2^n - 1 \)
5. \( a_n = 3n^2 + 1 \)
6. \( a_n = 3n^2 - 1 \)

Choose the word or phrase from the list that best completes each sentence.

9. An ordered list of numbers is called a(n) __________.
10. An ordered list of numbers is called a(n) __________.

Choose the word or phrase from the list that best completes each sentence.

11. \( a_n = 2n + 5 \) is a(n) __________.
12. \( a_n = 3n^2 + 1 \) is a(n) __________.

Find the eighth term of each sequence.

13. \( a_n = 2n + 1 \) \( a_8 = \) __________
14. \( a_n = 3n^2 + 1 \) \( a_8 = \) __________
15. \( a_n = 2^n + 1 \) \( a_8 = \) __________
16. \( a_n = 2^n - 1 \) \( a_8 = \) __________
17. \( a_n = 3n^2 + 1 \) \( a_8 = \) __________
18. \( a_n = 3n^2 - 1 \) \( a_8 = \) __________

Write a recursive definition for each sequence.

19. \( a_1 = 4, a_n = a_{n-1} + 2 \) \( a_2 = \) __________ \( a_3 = \) __________ \( a_4 = \) __________
20. \( a_2 = 3, a_n = 3a_{n-1} \) \( a_3 = \) __________ \( a_4 = \) __________ \( a_5 = \) __________
21. \( a_1 = 4, a_n = 2a_{n-1} \) \( a_2 = \) __________ \( a_3 = \) __________ \( a_4 = \) __________
22. \( a_1 = 1, a_n = a_{n-1} + 1 \) \( a_2 = \) __________ \( a_3 = \) __________ \( a_4 = \) __________
23. \( a_1 = 1, a_n = 2a_{n-1} \) \( a_2 = \) __________ \( a_3 = \) __________ \( a_4 = \) __________
24. \( a_1 = 2, a_n = a_{n-1} + 1 \) \( a_2 = \) __________ \( a_3 = \) __________ \( a_4 = \) __________
25. \( a_1 = 1, a_n = a_{n-1} \) \( a_2 = \) __________ \( a_3 = \) __________ \( a_4 = \) __________

Write an explicit formula for each sequence. Find the twentieth term.

26. \( a_n = 2n + 1 \) \( a_{20} = \) __________
27. \( a_n = 3n^2 + 1 \) \( a_{20} = \) __________
28. \( a_n = 2^n + 1 \) \( a_{20} = \) __________
29. \( a_n = 2^n - 1 \) \( a_{20} = \) __________
30. \( a_n = 3n^2 - 1 \) \( a_{20} = \) __________
31. \( a_n = 3n^2 + 1 \) \( a_{20} = \) __________

Find the first six terms of each sequence.

32. \( a_n = 3n^2 - 1 \)
33. \( a_n = 2^n - 1 \)
34. \( a_n = 3n^2 + 1 \)
35. \( a_n = 2^n + 1 \)
36. \( a_n = 2^n - 1 \)
37. A man swims 1.5 mi on Monday, 1.6 mi on Tuesday, 1.8 mi on Wednesday, 2.0 mi on Thursday, and 2.5 mi on Friday. If the pattern continues, how many miles will he swim on Saturday? 3.0 mi

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Write a recursive definition for each sequence.

7. The first five terms are 3, 7, 11, 15, 19.
   - Start with 1 and add 4 each time. $a_n = a_{n-1} + 4$

8. The first five terms are 2, 5, 8, 11, 14.
   - Start with 2 and add 3 each time. $a_n = a_{n-1} + 3$

Write a recursive definition for each sequence.

9. The first five terms are 2, 5, 8, 11, 14.
   - Start with 2 and add 3 each time. $a_n = a_{n-1} + 3$

10. The first five terms are 5, 10, 15, 20, 25.
    - Start with 5 and add 5 each time. $a_n = a_{n-1} + 5$

11. The first five terms are 9, 18, 27, 36, 45.
    - Start with 9 and add 9 each time. $a_n = a_{n-1} + 9$

12. Writing Explain the difference between a recursive definition and an explicit formula.
    - A recursive formula defines a sequence by the relationship between successive terms.

For Exercises 1−6, choose the correct letter.

5. Consider the sequence $a_1 = 2$, $a_2 = 5$, $a_3 = 8$, $a_4 = 11$, $a_5 = 14$, $a_n = a_{n-1} + 3$
   - The formula $a_n = a_{n-1} + 3$ best represents which sequence?

6. Consider the sequence $a_1 = 2$, $a_2 = 5$, $a_3 = 8$, $a_4 = 11$, $a_5 = 14$, $a_n = a_{n-1} + 3$
   - The formula $a_n = a_{n-1} + 3$ best represents which sequence?

7. Consider the sequence $a_1 = 2$, $a_2 = 5$, $a_3 = 8$, $a_4 = 11$, $a_5 = 14$, $a_n = a_{n-1} + 3$
   - The formula $a_n = a_{n-1} + 3$ best represents which sequence?

8. Consider the sequence $a_1 = 2$, $a_2 = 5$, $a_3 = 8$, $a_4 = 11$, $a_5 = 14$, $a_n = a_{n-1} + 3$
   - The formula $a_n = a_{n-1} + 3$ best represents which sequence?

9. Consider the sequence $a_1 = 2$, $a_2 = 5$, $a_3 = 8$, $a_4 = 11$, $a_5 = 14$, $a_n = a_{n-1} + 3$
   - The formula $a_n = a_{n-1} + 3$ best represents which sequence?

10. Consider the sequence $a_1 = 2$, $a_2 = 5$, $a_3 = 8$, $a_4 = 11$, $a_5 = 14$, $a_n = a_{n-1} + 3$
    - The formula $a_n = a_{n-1} + 3$ best represents which sequence?

11. Consider the sequence $a_1 = 2$, $a_2 = 5$, $a_3 = 8$, $a_4 = 11$, $a_5 = 14$, $a_n = a_{n-1} + 3$
    - The formula $a_n = a_{n-1} + 3$ best represents which sequence?

12. Consider the sequence $a_1 = 2$, $a_2 = 5$, $a_3 = 8$, $a_4 = 11$, $a_5 = 14$, $a_n = a_{n-1} + 3$
    - The formula $a_n = a_{n-1} + 3$ best represents which sequence?

Practice (continued) Form K

9.1 Enrichment Mathematical Patterns

Mathematical Patterns

You can define the terms in a sequence using an explicit formula or a recursive definition. You can use another method, called iteration, to form a sequence. The word iteration means to repeat an action. In mathematics, a sequence of numbers is generated through iteration when the same procedure is performed on each output.

10. Reasoning You and your friend are trying to find the 80th term in the sequence 8, 14, 20, 26, 32.
    - You use a recursive definition and your friend uses an explicit formula. Who will find the 80th term first? Why?
    - Your friend will find the 80th term first because he is using an explicit formula. Your friend will substitute 80 into the formula to get the answer, while you will go through 79 iterations of the recursive formula.

20. Your neighbor recently began learning to play the piano. On the first day, she practiced for 0.4 h. On the second day, she practiced for 0.5 h. She practiced for 0.65 h on the third day, and 0.85 h on the fourth day. If this pattern continues, how long will she practice on the seventh day? 1.75 h

21. Charles lost two rented movies, so he owes the rental store a fee of $40. At the end of each month, the amount that Charles owes will increase by 5%, plus a $2 billing fee. How much money will Charles owe the rental store after 8 months? $78.20

ANSWERS

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11. Each term is multiplied by 10. In each term, the number is increased by two more than the previous term; 33, 45, 59, ...

1. In each term, the number that is added can help with unfamiliar patterns.

In each term, the number that is added to get the next term increases by one. If the pattern is continued, the next term is 20 or 20.

Exercises
Describe the pattern that is formed. Find the next three terms.
1. 5, 6, 6, 11, 15, ...
   a) 2, 3, 6, 12, 24, 48
   b) 2, 5, 8, 11, 14, ...
   c) not arithmetic
   d) arithmetic

2. 1, 3, 9, 27, 81, ...
   a) 1, 3, 9, 27, 81
   b) 100, 90, 80, 70, 60
   c) 10, 10, 21, 24, 27
   d) 7, 5, 25, 125, 625, 3125

3. Each term is multiplied by 5 to get the next term; 15, 625; 78, 125; 390, 625
4. Each term is increased by 3 to get to the next term; 30, 33, 36
5. Each term is increased by 2 to get the next term; 75, 76, 78, 80, 82, ...
6. Each term is divided by 2 to get the next term; 7.5, 3.75, 1.875
7. Each term is divided by 2 and then 3 is added to get the next term; 125, 253, 505

Arithmetic Mean
The arithmetic mean is the average of a set of numbers. The arithmetic mean of two numbers and is found using the formula displayed below.
\[
\text{Mean} = \frac{a + b}{2}
\]

Sample
The arithmetic mean of 4 and 6 is \(\frac{4 + 6}{2} = \frac{10}{2} = 5\).

Find the missing number in the arithmetic sequence. This number is the arithmetic mean of the two given numbers.
8. . . . 13, 27, . . . .
9. . . . 30, 42, . . . .
10. . . . 45, 98, . . . .

ANSWERS

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1. Determine whether each sequence is arithmetic. If so, identify the common difference.
   1. 2, 3, 5, 8, 12 No
   2. 2, 4, 6, 8, 10 Yes; 2
   3. 9, 10, 11, 12, 13 Yes; 1
   4. 1, 3, 5, 7, 9 Yes; 2
   5. 14, 16, 18, 20, 22 Yes; 2
   6. 33, 34, 35, 36, 37 Yes; 1
   7. 12, 13, 14, 15, 16 Yes; 1
   8. 82, 83, 84, 85, 86 Yes; 1
   9. 127, 128, 129, 130, 131 Yes; 1

   Find the 10th term of each sequence.
   1. 2, 4, 6, 8, 10 10
   2. 13, 14, 15, 16, 17 17
   3. 22, 23, 24, 25, 26 26
   4. 127, 128, 129, 130, 131 131

   Find the missing term of each arithmetic sequence.
   1. 2, 4, 6, 8, 10 10
   2. 13, 14, 15, 16, 17 17
   3. 22, 23, 24, 25, 26 26
   4. 127, 128, 129, 130, 131 131

   Find the arithmetic mean of the given terms.
   1. 3, 5, 7 5
   2. 4, 6, 8, 10 7
   3. 9, 10, 11, 12 10

   Find the first five terms of each arithmetic sequence.
   1. 3, 5, 7, 9, 11 11
   2. 4, 6, 8, 10, 12 12

   Error Analysis
   Noah used the formula \( a_n = a_1 + (n-1)d \) to find the 12th term in the sequence 2, 4, 6, 8, 10, 12. Did Noah find the correct term? How do you know? No. Noah applied the explicit formula for arithmetic sequences to a sequence that is not arithmetic.
3. Find the 25th term of each sequence.

Exercises
Check the answer. Write 20, 16.5, and 13.

9-2

Enrichment
Arithmetic Sequences

There are many types of sequences. One interesting type of sequence is the Farey sequence. The first four Farey sequences are:

\[ F_1 = \{ \frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \ldots \} \]
\[ F_2 = \{ \frac{1}{2}, \frac{2}{3}, \frac{1}{3}, \frac{3}{5}, \frac{1}{4} \} \]
\[ F_3 = \{ \frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{1}{4}, \frac{4}{7}, \frac{1}{5}, \frac{1}{6}, \frac{2}{7}, \frac{3}{7}, \frac{1}{7} \} \]

Each Farey sequence is a list of fractions in increasing order between 0 and 1, written in simplest form with a denominator less than or equal to the integer \( n \).

For any \( n \) greater than 1, there are an odd number of terms in the sequence and the middle term is \( \frac{n}{n} \).

Problem
What are the terms of the Farey sequence for \( n = 5 \)?

The Farey sequence for \( n = 5 \) contains all the terms of the Farey sequence \( F_4 \) plus the fractions between 0 and 1 which have a denominator of 5 when written in simplest form.

The fractions \( \frac{5}{5} \) and \( \frac{0}{5} \) will not be added because they simplify to \( \frac{1}{1} \) and \( \frac{0}{1} \). Insert the fractions \( \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5} \) in the Farey sequence \( F_5 \):

\[ F_5 = \{ \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5} \} \]

Exercises
1. How many terms are in each of the first five Farey sequences? 2, 3, 5, 7, 11
2. What are the terms for the Farey sequence \( F_6 \)?
3. What will be the new terms in the Farey sequence \( F_{11} \) compared to the sequence \( F_{10} \)?
4. Since 11 is a prime number, how many more terms will be in the sequence \( F_{11} \) compared to the sequence \( F_{10} \)?
5. Is there any limit to how large \( n \) can be? No, \( n \) can be any positive integer although the computations become tedious.
6. Can you give examples of any other sequences?
   Answers may vary. Sample: arithmetic, geometric, and Fibonacci.
9-3  Ell Support
Geometric Sequences

Use the chart below to review vocabulary. These vocabulary words will help you complete this page.

<table>
<thead>
<tr>
<th>Vocabulary Words</th>
<th>Explanations</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometric sequence</td>
<td>A sequence in which the ratio of any term to its preceding term is a constant value.</td>
<td>The sequence 3, 6, 12, 24, ... is geometric because all of the consecutive terms have a ratio of 2.</td>
</tr>
<tr>
<td>Common ratio</td>
<td>The ratio of each term to its preceding term in a geometric sequence.</td>
<td>The common ratio in the sequence 1, 4, 16, 64, ... is 4.</td>
</tr>
<tr>
<td>Geometric mean</td>
<td>The geometric mean of two numbers and is ( \sqrt{xy} ).</td>
<td>The geometric mean of the numbers 4 and 9 is ( \sqrt{36} = 6 ).</td>
</tr>
</tbody>
</table>

1. The terms in the sequence 2, 6, 18, 54, ... all share a common ratio.
2. The terms in the sequence 2, 6, 18, 54, ... all share a common ratio.
3. The consecutive terms in a geometric sequence all share a common ratio.

Identify each sequence as arithmetic or geometric.
1. 2, 4, 8, 16, 32, ... geometric
2. 1, 3, 9, 27, ... geometric
3. 1, 4, 7, 10, ... arithmetic

Identify the common ratio for each geometric sequence.
7. 3, 12, 48, ... 4
8. 12, 36, 108, ... 3
9. Find the missing term in the geometric sequence.
   a. 5, 15, ... 45
   b. 9, 3, ... 1
   c. 50, 100, 200, ... 400

9-3  Think About a Plan
Geometric Sequences

Athletics During your first week of training for a marathon, you run a total of 10 miles. You increase the distance you run each week by twenty percent. How many miles do you run during your tenth week of training?

Understanding the Problem
1. How can you write a sequence of numbers to represent this situation?
   Answers may vary. Sample: Start with 10, and multiply it by each successive term by 120% or 5.2

2. Is the sequence arithmetic, geometric, or neither? geometric
3. What is the first term of the sequence? 10
4. What is the common ratio of the sequence? 1.2
5. What is the problem asking you to determine?
   the 12th term of a geometric sequence that represents the number of miles you run each week

Planning the Solution
6. Write a formula for the sequence.
   \( a_n = 50(1.2)^{n-1} \)

Getting an Answer
7. Evaluate your formula to find the number of miles you run during your tenth week of training.
   about 74.3

9-3  Practice
Geometric Sequences

Determine whether each sequence is geometric. If so, find the common ratio.
1. 1, 3, 9, 27, 81, 243, ... yes; 3
2. 2, 4, 8, 16, 32, ... yes; 2
3. 3, 6, 12, 24, 48, ... yes; 2
4. 4, 8, 16, 32, 64, ... yes; 2
5. 1, 3, 9, 27, ... yes; 3
6. 1, 4, 7, 10, ... no

Find the tenth term of each geometric sequence.
10. 2, 4, 8, 16, 32, ... 1024
11. 1, 3, 9, 27, 81, 243, ... 13,182
12. 2, 6, 18, 54, 162, ... 59,049
13. 3, 9, 27, 20, 63, 189, ... 19,683
14. 1, 2, 4, 8, 16, ... 256
15. 2, 4, 8, 16, 32, ... 512
16. 1, 4, 9, 16, ... 128
17. 1, 6, 27, 153, 855, ... 52,025
18. \( 1, 3, 9, 27, ... \)
19. When a pendulum is swung freely, the length of its arc decreases geometrically.
   Find each missing arc length.
   a. 20th arc is 10 in.; 22nd arc is 18 in. about 12 in.
   b. 6th arc is 27 mm, 10th arc is 33 mm. 9 mm
   5th arc is 25 mm, 7th arc is 41 mm 5 mm
d. 100th arc is 100 in. 99th arc is 8 8 in.
20. Find the missing term of each geometric sequence. It could be the geometric mean or its opposite.
   a. 4, 16, 64, ... 256
   b. 2, 8, 32, ... 128
21. Writing Explain how you know that the sequence 40, 80, 160, 320 is geometric.
   The sequence has a common ratio of 2 or 0.5 between terms.
22. Open-Ended Write a geometric sequence of at least seven terms.
   any geometric sequence with a common ratio
23. Error Analysis A student says that the geometric sequence 30, 120 can be completed with 90. Is she correct? Explain.
   No; the sequence can be completed with 40 with a common ratio of 2.

9-3  Practice (continued)
Geometric Sequences

Identify each sequence as arithmetic, geometric, or neither. Then find the next two terms.
32. 9, 18, 36, 72, ... arithmetic; 144, 288
33. 2, 2, 2, ... geometric; 2, 2
34. 3, 3, 3, ... geometric; 3, 3
35. 1, 5, 15, 45, ... arithmetic; 15, 45
36. 4, 12, 36, 108, ... arithmetic; 4, 4
37. 25, 50, 100, 200, ... arithmetic; 2, 2

Write an explicit formula for each sequence. Then generate the first five terms.
38. \( a_n = 3n - 2 \)
39. \( a_n = -3n + 5 \)
40. \( a_n = -3n + 8 \)
41. \( a_n = 3n - 2 \)
42. \( a_n = -n^2 + 3 \)
43. \( a_n = 2^n + 1 \)
44. \( a_n = 3n - 2 \)
45. \( a_n = -3n - 5 \)
46. \( a_n = -3n + 2 \)
47. The deer population in an area is increasing. This year, the population was 1,025. Last year’s population was 925.
   a. Assuming that the population increases at the same rate for the next few years, write an explicit formula for the sequence. \( a_n = 2037(1.025)^{n-1} \)
   b. Find the expected deer population for the fourth year of the sequence. about 2372
48. You expand the dimensions of a picture to 110% several times. After the first increase, the picture is 1 in. wide.
   a. Write an explicit formula to model the width after each increase. \( a_n = 1(1.1)^{n-1} \)
   b. How wide is the photo after the 2nd increase? 1.1 in.
   c. How wide is the photo after the 3rd increase? 2.2 in.
   d. How wide is the photo after the 12th increase? about 60.5 in.

Find the missing terms of each geometric sequence. (Hint: The geometric mean of positive first and fifth terms is the third term. Some terms might be negative.)
49. 2, 6, 18, 54, ... 162, 486
50. 3, 18, 108, 648, 3888
51. 3, 3, 3, 3, 3, 3, 3, 3, ... arithmetic; 3, 3

For the geometric sequence 6, 18, 54, 162, ... find the indicated term.
51. 6th term 1628
52. 11th term 2,324,522,934
53. nth term \( 6(3^{n-1}) \)

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10. Find the missing terms of each geometric sequence.

6. 2, ___, ___, 128, ___.
7. 3, 12, 48, 192, ___.
8. 1, 3, 9, 27, ___.

Open-Ended

Identify the common ratio.

What is the doubling period for a geometric sequence greater than 1?

Error Analysis

On a recent math test, your classmate was asked to find the missing term in the geometric sequence 4, ___, 216. Her answer was 136. What error did your classmate make? What is the correct answer?

A corporation earned a profit of $420,000 in its first year of operation. Over the next 10 years, the company’s CEO hopes to increase the profit by 8% each year. If the CEO reaches her goal, what will be the company’s profit in its seventh year, to the nearest dollar? $566,487
9-3 Reaching Geometric Sequences

- A geometric sequence has a constant ratio between consecutive terms. This number is called the common ratio.
- A geometric sequence can be described by a recursive formula, \(a_n = a_{n-1} \cdot r\), or as an explicit formula, \(a_n = a_1 \cdot r^{n-1}\).

**Problem**
Find the 12th term of the geometric sequence 5, 15, 45, 

\[ r = \frac{15}{5} = 3 \]

Find the common ratio between consecutive terms. This is a geometric sequence because there is a common ratio between consecutive terms.

\[ a_1 = 5; \quad a_{12} = 5(3)^{11} \]

Substitute \( n = 12 \) to find the 12th term of the sequence.

\[ a_{12} = 885,735 \]

Remember to first calculate \( 3^{11} \), then multiply by 5.

**Exercises**

Find the indicated term of the geometric sequence.

1. \( a_1, a_2, \dotsc \) Find \( a_5 \) \( \frac{1}{2}, \frac{1}{6}, \frac{1}{18}, \frac{1}{54} \) \( a_5 = \frac{1}{2}(3)^4 = \frac{1}{162} \)
2. \( a_1, a_2, \dotsc \) Find \( a_6 \) \( 1, 2, 4, 8, 16, 32 \) \( a_6 = 1(2)^5 = 32 \)

Write the explicit formula for each sequence. Then generate the first five terms.

7. \( a_1 = 2; \quad a_2 = 2 + 1 \)

\[ a_1 = 2, \quad a_2 = 2 + 1, \quad a_3 = 2 + 2, \quad a_4 = 2 + 3, \quad a_5 = 2 + 4 \]

8. \( a_1 = 1, \quad a_2 = 1 \cdot 5, \quad a_3 = 1 \cdot 5^2, \quad a_4 = 1 \cdot 5^3, \quad a_5 = 1 \cdot 5^4 \)

Find the total seating capacity of the theater.

- In a 20-row theater, the number of seats in a row increases by three.
- Architecture

**9-4 Think about a Plan**

**Arithmetic Series**

- In a 20-row theater, the number of seats in a row increases by three with each successive row. The first row has 18 seats.

\[ a_1 = 18; \quad a_2 = 21; \quad a_3 = 24; \quad \ldots \]

- Write an arithmetic series to represent the number of seats in the theater.

- Find the total seating capacity of the theater.

- Front-row tickets for a concert cost $80. After every 5 rows, the ticket price goes down by $5. What is the total amount of money generated by a full house?

\[ \sum (80 + 5n) \]

- Write an explicit formula for the arithmetic sequence that represents the number of seats in each row.

\[ a_n = a_1 + (n - 1)d \]

- Write an arithmetic series to represent the number of seats in the theater.

- How can you use a graphing calculator to evaluate the series?

**Answers may vary. Use the sum command and the sequence command number(20 + 5n), 1, 20)\n
- Find the total seating capacity of the theater.

**Step 1** Identify key information in the problem.
- Know that your friend’s rent was $1000 in 2000. This means \( a = 1000 \). You also know that her rent in 2003 was $1062.73. This means that \( a_3 = 1062.73 \). Her rent is raised by the same percent each year, which is the same as multiplying by a constant (e.g., a 5% increase is the same as multiplying by 1.05).

**Step 2** Identify missing information.
- You need to find the common ratio to in order to find the rent in 2009, \( a_{10} \)

**Step 3** Use the explicit formula to find \( a_n \)

\[ a_n = a_1 + (n - 1)d \]

Write the explicit formula.

\[ a_n = 1000(1.05)^{n-1} \]

Substitute \( n = 10 \) and \( n = 10 \).

\[ a_{10} = (1000)(1.05)^{10-1} \]

Simplify.

\[ a_{10} = 1315.78 \]

Compute Round to the nearest hundredth.

Your friend’s rent was $1315.78 in 2009.

**Exercises**

22. An athlete is training for a bicycle race. She increases the amount she bikes by the same percent each day. If she bikes 10 mi on the first day, and 12.1 mi on the third day, how much will she bike on the fifth day? Use what percent does she increase the amount she bikes each day? \( 14.641 \text{ mi} \)

23. By clipping coupons and using more meals at home, your family plans to decrease their monthly food budget by the same percent each month. If they budgeted $600 in January and $514.43 in April, how much will they budget in December? \$431.28

24. From 2005 to 2009, a teen raised her babysitting rate by a fixed percent every year. If she charged $8.00 in 2005 and $18.04 in 2007, how much did she charge in 2009? What is her percent of increase each year? \$10.04/yr; 12%
9.4 Practice

Form G

Arithmetic Series

Find the sum of each finite arithmetic series.

1. \(a_1 = 5 \quad d = 2 \quad a_n = 29\) 
2. \(a_1 = 4 \quad d = 3 \quad a_n = 41\)
3. \(a_1 = 2 \quad d = 5 \quad a_n = 47\)
4. \(a_1 = 10 \quad d = 3 \quad a_n = 50\)
5. \(a_1 = 15 \quad d = 4 \quad a_n = 65\)
6. \(a_1 = 20 \quad d = 5 \quad a_n = 75\)

Find the number of terms.

7. \(a_1 = 3 \quad d = 7 \quad a_n = 25\) 
8. \(a_1 = 4 \quad d = 8 \quad a_n = 36\)
9. \(a_1 = 5 \quad d = 9 \quad a_n = 45\)
10. \(a_1 = 6 \quad d = 10 \quad a_n = 55\)

Find the sum.

11. \(a_1 = 7 \quad d = 2 \quad a_n = 21\) 
12. \(a_1 = 8 \quad d = 3 \quad a_n = 28\)
13. \(a_1 = 9 \quad d = 4 \quad a_n = 36\)
14. \(a_1 = 10 \quad d = 5 \quad a_n = 45\)

Write each arithmetic series in summation notation.

15. \(a_n = 2n + 1\) 
16. \(a_n = 3n - 2\)
17. \(a_n = 4n + 3\) 
18. \(a_n = 5n - 2\)
19. \(a_n = 6n + 1\) 
20. \(a_n = 7n - 3\)

Find the sum of each infinite series.

21. \(\sum_{n=1}^{\infty} (n-1)\) 
22. \(\sum_{n=1}^{\infty} (2n+3)\)
23. \(\sum_{n=1}^{\infty} (3n-4)\) 
24. \(\sum_{n=1}^{\infty} (4n+2)\)
25. \(\sum_{n=1}^{\infty} (5n-1)\)
26. \(\sum_{n=1}^{\infty} (6n+3)\)

ANSWERS

9.4 Practice (continued)

Form G

Arithmetic Series

Determine whether each list is a sequence or a series and finite or infinite.

27. \(7, 14, 21, 28\) 
28. \(10, 20, 30, 40\)
29. \(1, 3, 5, 7, 9, 11\) 
30. \(2, 4, 6, 8, 10, 12\)

Write each arithmetic series in summation notation.

31. \(a_1 = 2 \quad d = 2 \quad a_n = 20\) 
32. \(a_1 = 3 \quad d = 3 \quad a_n = 30\)
33. \(a_1 = 4 \quad d = 4 \quad a_n = 40\) 
34. \(a_1 = 5 \quad d = 5 \quad a_n = 50\)

Write three explicit formulas for arithmetic sequences.

35. \(a_n = 2n + 1\) 
36. \(a_n = 3n - 2\)
37. \(a_n = 4n + 3\) 
38. \(a_n = 5n - 2\)

39. \(a_n = 6n + 1\) 
40. \(a_n = 7n - 3\)

41. \(a_n = 8n + 2\) 
42. \(a_n = 9n - 4\)

43. \(a_n = 10n + 0\) 
44. \(a_n = 11n - 8\)

45. \(a_n = 12n - 16\) 
46. \(a_n = 13n - 24\)

47. \(a_n = 14n - 32\) 
48. \(a_n = 15n - 40\)

49. \(a_n = 16n - 48\) 
50. \(a_n = 17n - 56\)

51. \(a_n = 18n - 64\) 
52. \(a_n = 19n - 72\)

53. \(a_n = 20n - 80\) 
54. \(a_n = 21n - 88\)

55. \(a_n = 22n - 96\) 
56. \(a_n = 23n - 104\)

57. Your brother is preparing for basketball season. He shot 26 baskets on the first day that he practiced. He shot 32 baskets on the second day and 38 baskets the day after that.

a. If this pattern continues, how many baskets will he shoot on the 30th day?

b. How many baskets will he have shot during those 30 days?

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9-4 Standardized Test Prep
Arithmetic Series

Multiple Choice
For Exercises 1–6, choose the correct letter.
1. What is the sum of the first 100 even positive integers? B
   A) 100 B) 10,000 C) 1000 D) 5050

2. Which of the following is an infinite series? H
   A) 3 + 8 + 13 + 18 + 23 + ... B) 5 + 10 + 15 + 20 + 25 + ... 
   C) 3, 8, 13, 18, 23, ... D) 3 + 8 + 13 + 18 + 23 + ...

3. The high school choir is participating in a fundraising sales contest. The choir
   will receive a bonus if they make 20 sales in their first week and improve their
   sales by 5 in every subsequent week. What is the minimum number of sales
   the choir could make in the first 12 weeks to qualify for the bonus? C
   A) 13 B) 53 C) 438 D) 5015

4. What is summation notation for the series 5 + 7 + 9 + ... + 105? F
   \( \sum_{n=1}^{15} (2n+3) \)

5. What is the upper limit of the summation \( \sum_{n=1}^{10} (n-2)^3 \)? D
   A) 1 B) 2 C) 9 D) 100

6. What is the sum of the series \( \sum_{n=1}^{10} 2n+1 \)? H
   A) 62 B) 66 C) 990 D) 1980

Short Response
7. What is the sum of the first finite series 2 + 4 + 6 + ... + 50? Show your work.
   \( S = \frac{n(a_1 + a_n)}{2} \) where \( n = 26, a_1 = 2, a_n = 50 \)
   \( S = \frac{26(2 + 50)}{2} = 13 \times 52 = 650 \)

9-4 Enrichment
Arithmetic Series

The Gauss Trick
If your teacher asked you to add the numbers from 1 to 100, you would probably begin by adding
1 + 2 + 3 + ... + 100, term by term from left to right. Carl Friedrich Gauss (1777–
1855) had a different way. Let’s represent the finite series whose sum you are trying to find.
Since addition is commutative, both equations below represent this series.
\( S = 1 + 2 + 3 + 4 + ... + 99 + 100 \)
\( S = 100 + 99 + 98 + ... + 3 + 2 + 1 \)
1. What is the sum of the left side of the first equation and the left side of the second equation? 2n
2. What is the sum of each vertically aligned pair of quantities on the right side of the equal signs? 100
3. How many such pairs are there? 100
4. Because each pair has the same sum, use multiplication to express the sum of all the pairs on the right side.
   \( S = \frac{n(n + 1)}{2} \) for all \( n \)
5. Write an equation that states that the sum of the left sides equal the sum of the right sides. Solve your equation for \( S \).
   \( 2S = n(n + 1) \)
   \( S = \frac{n(n + 1)}{2} \)

Use the technique outlined above to derive the formula for the sum of \( n \) terms of any
arithmetic series. Suppose that the series starts with the term \( a_1 \) and has a common
difference of \( d \).
6. What is the \( n \)th term, in terms of \( a_1 \) and \( d \)?
   \( a_n = a_1 + (n - 1)d \)

7. Write the sum of the \( n \) terms of the series, where each number is written in
terms of \( a_1 \) and \( d \). Then write the sum in reverse order, lining up terms.
   \( S = a_1 + (a_1 + d) + (a_1 + 2d) + ... + (a_1 + (n - 2)d) + (a_1 + (n - 1)d) \)
   \( S = (a_1 + (n - 1)d) + (a_1 + (n - 2)d) + ... + (a_1 + d) + a_1 \)
8. What is the sum of each vertically aligned pair of quantities in the right side?
   \( 2S = (n + 1)(a_1 + a_n) \)
9. How many such pairs are there? \( n \)
10. Express the sum of all the pairs using multiplication.
   \( S = \frac{n}{2} [2a_1 + (n - 1)d] \)
   Use the answer to Exercise 6.
   \( S = \frac{n}{2} [2a_1 + (n - 1)d] \)

9-4 Retracing
Arithmetic Series

Summation notation shows the upper limit, lower limit, and explicit formula for
the terms of a series.
To find the sum of an arithmetic series written in summation notation:
• list the terms and add them, or use the formula
   \( S_n = \frac{n(a_1 + a_n)}{2} \)

Problem
What is the sum of the series written in summation notation?
\( \sum_{n=1}^{4} (2n - 1) \)

Exercise
Find the sum of each finite series.
1. \( \sum_{n=1}^{10} (n^2 - 2n) \)
2. \( \sum_{n=1}^{10} (n + 1) \)
3. \( \sum_{n=1}^{10} (2n - 1) \)
4. \( \sum_{n=1}^{10} (3n - 2) \)

5. \( \sum_{n=1}^{20} (2n - 1) \)
6. \( \sum_{n=1}^{15} (n^2 - 2n) \)
7. \( \sum_{n=1}^{10} (n^2 - 2n) \)
8. \( \sum_{n=1}^{15} (2n - 1) \)
10. What is the sum of the geometric series \( 2 + 6 + 18 + 54 + \cdots = 1458 \)?

11. Identify the common ratio and the nth term.

12. Identify the first term, common ratio, and nth term.

13. Use the explicit formula.

14. Divide each side by 2.

15. Use the sum formula.

16. Substitute 1 for \( r \).

17. Use the sum formula.

18. Use a calculator.

19. Substitute 1 for \( r \) and 7 for \( n \).

20. The company expects its sales to drop 10% each succeeding year. Find the total sales.

21. The end of a pendulum travels 50 cm on its first swing. Each swing after the first, it travels 99% as far as the preceding swing. How far will the pendulum travel in its first six stages?

22. The first year a toy manufacturer introduces a new toy, its sales total $495,000. The sales increased 20% per year, find the total profit over the first 5 yr.

23. The company expects its sales to drop 10% each succeeding year. Find the total sales.

24. Identify the first term, common ratio, and nth term.

25. Use the sum formula.

26. Do your answer agree with your sum from Exercise 5?

27. What type of diagram can you make to represent the telephone chain?

28. Write the series that represents the total number of calls made through the first six stages.

29. How many employees have been notified after stage six?

30. Write the series that represents the total number of calls made through the first six stages.

31. How many employees have been notified after stage six?

32. Write an infinite geometric series that converges to 2.

33. Does your answer agree with your sum from Exercise 5?

34. Choose the correct choice below.

35. Identify the first term, common ratio, and nth term.

36. Use the sum formula.

37. What type of diagram can you make to represent the telephone chain?

38. Use the sum formula.
Find the sum of each finite geometric series.

1. \[2 + 6 + 18 + \ldots + 4374\]
   - Find the number of terms: \[n = \frac{4374 - 2}{6} + 1 = 799\] 
   - Use the sum formula: \[S_n = \frac{a_1(1 - r^n)}{1 - r}\] 
   - Substitute: \[S_n = \frac{2(1 - 3^{799})}{1 - 3} = 3^{800} - 2\]

2. \[1 + 2 + 4 + \ldots + 2048\]
   - Use the sum formula: \[S_n = \frac{a_1(1 - r^n)}{1 - r}\] 
   - Substitute: \[S_n = \frac{1(1 - 2^{11})}{1 - 2} = 2^{11} - 1\]

3. \[B + 4 + 8 + 16 + \ldots + \frac{1}{16}\]
   - Use the sum formula: \[S_n = \frac{a_1(1 - r^n)}{1 - r}\] 
   - Substitute: \[S_n = \frac{1(1 - 2^{-5})}{1 - 2} = 2^{-5} - 1\]

4. \[5 - 8 - 16 - \ldots - 2048\]
   - Use the sum formula: \[S_n = \frac{a_1(1 - r^n)}{1 - r}\] 
   - Substitute: \[S_n = \frac{5(1 - (-2)^4)}{1 - (-2)} = -\frac{5}{3}(2^8 - 1)\]

5. \[7 + 3 + 1 + \ldots + \frac{1}{5}\]
   - Use the sum formula: \[S_n = \frac{a_1(1 - r^n)}{1 - r}\] 
   - Substitute: \[S_n = \frac{7(1 - \frac{1}{5})}{1 - \frac{1}{5}} = 5\]

6. \[3 + 6 + 12 + 24 + \ldots + 4867\]
   - Use the sum formula: \[S_n = \frac{a_1(1 - r^n)}{1 - r}\] 
   - Substitute: \[S_n = \frac{3(1 - 2^5)}{1 - 2} = 3(32 - 1)\]

7. A family farm produced 2400 ears of corn in its first year. For each of the next 9 yr, the farm increased its yearly corn production by 15%. How many ears of corn did the farm produce during this 10-yr period? \[48,729\]

Find the number of terms. Use the sum formula.

1. \[1 + \frac{1}{4} + \frac{1}{16} + \ldots + \frac{1}{512}\]
   - Use the sum formula: \[S_n = \frac{a_1(1 - r^n)}{1 - r}\] 
   - Substitute: \[S_n = \frac{1(1 - \frac{1}{2^9})}{1 - \frac{1}{2}} = 2(1 - \frac{1}{512})\]

2. \[\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{256}\]
   - Use the sum formula: \[S_n = \frac{a_1(1 - r^n)}{1 - r}\] 
   - Substitute: \[S_n = \frac{1/2(1 - \frac{1}{2}^{-7})}{1 - \frac{1}{2}} = 2(1 - \frac{1}{128})\]

3. \[\frac{1}{3} + \frac{1}{6} + \frac{1}{12} + \ldots + \frac{1}{120}\]
   - Use the sum formula: \[S_n = \frac{a_1(1 - r^n)}{1 - r}\] 
   - Substitute: \[S_n = \frac{1/3(1 - \frac{1}{4}^{-3})}{1 - \frac{1}{4}} = \frac{4}{3}(1 - \frac{1}{64})\]

4. \[\frac{1}{5} + \frac{1}{10} + \frac{1}{20} + \ldots + \frac{1}{100}\]
   - Use the sum formula: \[S_n = \frac{a_1(1 - r^n)}{1 - r}\] 
   - Substitute: \[S_n = \frac{1/5(1 - \frac{1}{2}^{-2})}{1 - \frac{1}{2}} = \frac{2}{5}(1 - \frac{1}{4})\]

5. \[\frac{1}{7} + \frac{1}{14} + \frac{1}{28} + \ldots + \frac{1}{112}\]
   - Use the sum formula: \[S_n = \frac{a_1(1 - r^n)}{1 - r}\] 
   - Substitute: \[S_n = \frac{1/7(1 - \frac{1}{2}^{-3})}{1 - \frac{1}{2}} = \frac{2}{7}(1 - \frac{1}{8})\]

Answers

1. 10
2. 10
3. 10
4. 10
5. 10
5. Find the first five terms of each sequence.

- Open-Ended
  
  a. 4, 12, 36, 108, 

  b. 13, 19, 25, 31, 

  c. Simplify inside the parentheses.
  
  d. Geometric Series
  
  e. yes;
  
  f. The cost of hosting the reunion in 2005 was $896.

Exercises

1. Solve for the specified number of terms.

- Geometric series
  
  a. 1, 3, 9, 27, 81, ... , n = 6
  
  b. 2, 4, 8, 16, 32, ... , n = 10

2. Simplify inside the parentheses.

- Geometric series
  
  a. 2, 6, 18, 54, 162, ... , n = 5

3. Find the fourth term of each sequence.

- 10, 20, 40, 80, 

- 5, 15, 45, 135, 

- Determine whether each sequence is arithmetic. If so, identify the common difference.

- 5, 13, 21, 29, 

- Find the missing term of each arithmetic sequence.

- 5, 3, 1, -1, 

- 2. Do you UNDERSTAND?

- 9. Vocabulary
  
  a. Explain what it means for a formula to be an explicit formula.

- 10. Open-Ended
  
  a. Give an example of an arithmetic sequence.

Do you know HOW?

Find the first five terms of each sequence.

1. a. 2

2. a. -5

3. a. 11

4. a. -11

5. a. 6

6. a. 54

Do you know HOW?

Find the eighth term of each geometric sequence.

1. 4, 8, 16, 

2. 20, 40, 80, 

3. Find the seventh term of each sequence.

1. 3, 6, 12, 24, 

2. 10, 9, 8, 7, 

3. Determine whether each sequence is arithmetic or geometric. If so, identify the common difference.

1. 1, 3, 5, 7, 

2. a. 5

3. a. 12

4. a. 1

5. a. 10

Do you UNDERSTAND?

9. Open-Ended

a. Write an arithmetic series that has a negative sum.

b. Determine if the series converges when the common ratio is greater than 1. Explain.

Do you know HOW?

Find the eighth term of each geometric sequence.

1. 4, 8, 16, 

2. 20, 40, 80, 

3. Find the seventh term of each sequence.

1. 3, 6, 12, 24, 

2. 10, 9, 8, 7, 

3. Determine whether each sequence is arithmetic or geometric. Then evaluate the finite series for the specified number of terms.

1. 1, 3, 5, 7, 

2. a. 5

3. a. 12

4. a. 1

5. a. 10

Do you UNDERSTAND?

9. Open-Ended

a. Write an arithmetic series that has a negative sum.

b. Determine if the series converges when the common ratio is greater than 1. Explain.

Do you know HOW?

Find the eighth term of each geometric sequence.

1. 4, 8, 16, 

2. 20, 40, 80, 

3. Find the seventh term of each sequence.

1. 3, 6, 12, 24, 

2. 10, 9, 8, 7, 

3. Determine whether each sequence is arithmetic or geometric. Then evaluate the finite series for the specified number of terms.

1. 1, 3, 5, 7, 

2. a. 5

3. a. 12

4. a. 1

5. a. 10

Do you UNDERSTAND?

9. Open-Ended

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b. Determine if the series converges when the common ratio is greater than 1. Explain.

Do you know HOW?

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1. 4, 8, 16, 

2. 20, 40, 80, 

3. Find the seventh term of each sequence.

1. 3, 6, 12, 24, 

2. 10, 9, 8, 7, 

3. Determine whether each sequence is arithmetic or geometric. Then evaluate the finite series for the specified number of terms.

1. 1, 3, 5, 7, 

2. a. 5

3. a. 12

4. a. 1

5. a. 10

Do you UNDERSTAND?

9. Open-Ended

a. Write an arithmetic series that has a negative sum.

b. Determine if the series converges when the common ratio is greater than 1. Explain.

Do you know HOW?

Find the eighth term of each geometric sequence.

1. 4, 8, 16, 

2. 20, 40, 80, 

3. Find the seventh term of each sequence.

1. 3, 6, 12, 24, 

2. 10, 9, 8, 7, 

3. Determine whether each sequence is arithmetic or geometric. Then evaluate the finite series for the specified number of terms.

1. 1, 3, 5, 7, 

2. a. 5

3. a. 12

4. a. 1

5. a. 10

Do you UNDERSTAND?

9. Open-Ended

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b. Determine if the series converges when the common ratio is greater than 1. Explain.

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Find the eighth term of each geometric sequence.

1. 4, 8, 16, 

2. 20, 40, 80, 

3. Find the seventh term of each sequence.

1. 3, 6, 12, 24, 

2. 10, 9, 8, 7, 

3. Determine whether each sequence is arithmetic or geometric. Then evaluate the finite series for the specified number of terms.

1. 1, 3, 5, 7, 

2. a. 5

3. a. 12

4. a. 1

5. a. 10

Do you UNDERSTAND?

9. Open-Ended

a. Write an arithmetic series that has a negative sum.

b. Determine if the series converges when the common ratio is greater than 1. Explain.

Do you know HOW?

Find the eighth term of each geometric sequence.

1. 4, 8, 16, 

2. 20, 40, 80, 

3. Find the seventh term of each sequence.

1. 3, 6, 12, 24, 

2. 10, 9, 8, 7, 

3. Determine whether each sequence is arithmetic or geometric. Then evaluate the finite series for the specified number of terms.

1. 1, 3, 5, 7, 

2. a. 5

3. a. 12

4. a. 1

5. a. 10

Do you UNDERSTAND?

9. Open-Ended

a. Write an arithmetic series that has a negative sum.

b. Determine if the series converges when the common ratio is greater than 1. Explain.

Do you know HOW?

Find the eighth term of each geometric sequence.

1. 4, 8, 16, 

2. 20, 40, 80, 

3. Find the seventh term of each sequence.

1. 3, 6, 12, 24, 

2. 10, 9, 8, 7, 

3. Determine whether each sequence is arithmetic or geometric. Then evaluate the finite series for the specified number of terms.

1. 1, 3, 5, 7, 

2. a. 5

3. a. 12

4. a. 1

5. a. 10

Do you UNDERSTAND?

9. Open-Ended

a. Write an arithmetic series that has a negative sum.

b. Determine if the series converges when the common ratio is greater than 1. Explain.

Do you know HOW?
Chapter 9 Quiz 1

Lessons 9.1 through 9.2

Do you know HOW?

Find the first four terms of each sequence.
1. \(a_n = 5n - 2\)
2. \(a_n = n^2 + 3\)
3. \(a_n = \frac{n}{n+2}\)

Write a recursive definition for each sequence.
4. \(4, 8, 16, 32, 64, \ldots\) 
5. \(5, 3, 11, 19, 27, \ldots\)
6. \(3, 20, 105, 525, \ldots\)

Find the 10th term of each arithmetic sequence.
7. \(a_1 = 2, d = 3\)
8. \(a_1 = 100, d = -5\)
9. \(a_1 = 3, d = 2\)

Use the arithmetic mean to find the missing term in each arithmetic sequence.
10. \(\ldots, 6, \ldots\)
11. \(\ldots, -14, \ldots\)
12. \(\ldots, 1.4, \ldots\)

Do you UNDERSTAND?

13. Writing Describe the difference between a recursive definition and an explicit definition of a sequence. An explicit definition relates each term to the next. An explicit definition describes the nth term of a sequence using the number n.

14. Tim takes the stairs up to his office. He enters the ground floor of the building and climbs 12 steps to reach the first floor. He climbs a total of 24 steps to reach the second floor and 36 steps to reach the third floor. How many steps will Tim climb to reach his office on the 10th floor? 152 steps.
Chapter 9 Performance Tasks

Task 1

a. Use the graphing calculator to graph the function \( f(x) = 2x \) over the domain \((-5, 5)\).

b. Use the TABLE feature on your calculator to calculate a table of values for the function from the set of values in part (a).

c. Determine whether the sequence of function values is arithmetic, geometric, or neither. Justify your answer.

d. Write a recursive definition and an explicit formula for the sequence of function values.

Task 2

a. Determine whether the sequence \( 2, 4, 8, 16, 32, \ldots \) is geometric, arithmetic, or neither. Justify your response.

b. Write a recursive definition and an explicit formula for this sequence.

c. Use summation notation to write the series related to the infinite sequence given in part (a). Then evaluate this series.

d. Use the summation notation to write the series related to the infinite sequence given in part (a). Determine whether this series diverges or converges. Explain.

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Chapter 9 Test

Do you know HOW?

Find the first five terms of each sequence.

1. \( a_n = 3n + 4 \)
   \( 7, 10, 13, 16, 19 \)
2. \( a_n = n^2 - 2 \)
   \( 2, 6, 12, 20, 30 \)
3. \( a_n = \sqrt{n} + 1 \)
   \( 1.5, 1, 0.5, 0.5 \)

Write an explicit formula for each sequence. Then find the 12th term.

4. \( a_n = 3, 5, 7, 9, 11, 13, \ldots \)
   \( a_n = 2n + 1 \)
5. \( a_n = n^2 + 2 \)
   \( 8 \)
6. \( a_n = 2, 5, 10, 17, 26, \ldots \)
   \( a_n = n^2 + 1; 145 \)

Find the 20th term of each arithmetic sequence.

\( 7, 5, 3, 1, -1, -3, \ldots \)
2. \( 8, 6, 4, 2, 0, -2, -4, \ldots \)
3. \( 2, 2.6, 3.4, 4.2, \ldots \)

Do you UNDERSTAND?

10. Writing Find the missing term in the sequence below. Explain how you found the term.

   \( \ldots, 12, \ldots, 24, \ldots, 44, \ldots \)

   Answers may vary. Sample: First, I found the sum of 12 and 44, which is 56. Then I divided the sum by 2 to get the missing term, 28.

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Chapter 9 Test

Do you know HOW?

Find the 9th term of each geometric sequence.

13. \( 5, 10, 20, 40, \ldots \)
   \( \frac{1}{2} \)
14. \( 32, -8, 2, -0.5, \ldots \)
   \( 2 \)
15. \( -2, -\frac{1}{2}, -\frac{1}{4}, -\frac{1}{8}, \ldots \)
   \( -\frac{1}{5} \)

Find the sum of each finite arithmetic series.

16. \( 8 + 10 + 12 + \ldots + 118 \)
   \( 1090 \)
17. \( -3 + (-9) + (-15) + \ldots + (-201) \)
   \( -4368 \)

18. \( 1 + 5 + 9 + \ldots + 157 \)
   \( 3160 \)

Do you UNDERSTAND?

22. Error Analysis Your friend calculated the sum of the finite geometric series

   \( 2 + 6 + 18 + \ldots + 3780 \)

   Her answer was 131,880. What error did she make?

   What is the correct sum?

   She used the formula for the sum of an arithmetic series rather than the sum of a geometric series; 63,600

23. Writing Find the possible values of the missing term in the following geometric sequence, and explain how you found the answer.

   \( 4, 2, 1, \ldots \)

   \( 2, 6, 18, 54, 162 \)

   \( 25, 5, 10, 20, \ldots \)

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Chapter 9 Performance Tasks

Task 3

a. Determine whether the sequence \( 2, 4, 8, 16, 32, \ldots \) is arithmetic or geometric, as neither.

b. Write a recursive definition and an explicit formula for this sequence.

\( a_n = 2a_{n-1}; a_1 = 2 \)
\( a_n = 2^{n-1} \)

(continued)

Chapter 9 Test

Do you know HOW?

Find the 9th term of each geometric sequence.

13. \( 5, 10, 20, 40, \ldots \)
   \( 1250 \)
14. \( 32, -8, 2, -0.5, \ldots \)
   \( \frac{1}{5} \)
15. \( -2, -\frac{1}{2}, -\frac{1}{4}, -\frac{1}{8}, \ldots \)
   \( -\frac{1}{5} \)

Find the sum of each finite arithmetic series.

16. \( 8 + 10 + 12 + \ldots + 118 \)
   \( 1090 \)
17. \( -3 + (-9) + (-15) + \ldots + (-201) \)
   \( -4368 \)

18. \( 1 + 5 + 9 + \ldots + 157 \)
   \( 3160 \)

Do you UNDERSTAND?

22. Error Analysis Your friend calculated the sum of the finite geometric series

   \( 2 + 6 + 18 + \ldots + 3780 \)

   Her answer was 131,880. What error did she make?

   What is the correct sum?

   She used the formula for the sum of an arithmetic series rather than the sum of a geometric series; 63,600

23. Writing Find the possible values of the missing term in the following geometric sequence, and explain how you found the answer.

   \( 4, 2, 1, \ldots \)

   \( 2, 6, 18, 54, 162 \)

   \( 25, 5, 10, 20, \ldots \)

Answers may vary. Sample: First, I found the product of 55 and 6, which is 570. Then I found the square root of 570, which is 24.

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Chapter 9 Performance Tasks

Task 4

a. Graph the function \( f(x) = -0.5x^2 + 3 \) for the domain \(-3 \leq x \leq 3\) using your graphing calculator.

b. Carefully draw the graph of the function on a sheet of graph paper. Check student's drawing.

c. Draw a unit square and use it to approximate the area under the curve for the given interval \(-3 \leq x \leq 3\).

\( \text{Area} \approx \) \text{Unit Square}
Multiple Choice

1. What is the y-intercept of \( y = 0.75x + 3 \)?
   - A. (0, 0)
   - B. (0, 3)
   - C. (0, 1)
   - D. (4, 0)
   - E. (4, 6)

2. What is \( \frac{x^2 - 2x - 8}{x - 2} \) simplified form?
   - A. \( x + 4 \)
   - B. \( x - 4 \)
   - C. \( 2x - 4 \)
   - D. \( x - 2 \)
   - E. \( x + 2 \)

3. Which expression is equivalent to \( \sqrt[3]{x^3} \)?
   - A. \( x^2 \)
   - B. \( x^3 \)
   - C. \( x \)
   - D. \( x + 2 \)
   - E. \( x - 2 \)

4. How is the polynomial \( 2x^2 - 3x + 1 \) classified by degree?
   - A. linear
   - B. quadratic
   - C. cubic
   - D. quartic

5. The discriminant of a quadratic equation has a value of 0. Which of the following might it represent?
   - A. Two real solutions.
   - B. One real solution.
   - C. Two complex solutions.
   - D. No real solutions.
   - E. Two solutions.

6. Which of these does not have the same value as the others?
   - A. \( \sqrt{16} \)
   - B. \( 2^4 \)
   - C. \( 2^2 \)
   - D. \( 4^2 \)
   - E. \( 16 \)

7. Which inequality is graphed?
   - A. \( y > x + 4 \)
   - B. \( y < x + 4 \)
   - C. \( y = x + 4 \)
   - D. \( y = 4x + 4 \)
   - E. \( y > 4x + 4 \)

8. If \( f(x) = 4x + 1 \) and \( g(x) = 2x^2 \), what is the value of \( f(2) / g(2) \)?
   - A. 0.5
   - B. 1
   - C. 2
   - D. 4
   - E. 8

9. Graph the system of inequalities.
   - A. \( y < 2x - 1 \)
   - B. \( y > x + 3 \)

10. Describe how the graph of \( y = \log_2(x - 2) + 5 \) compares to the graph of the parent function.
    - A. The graph of \( y = \log_2(x - 2) + 5 \) is a shift of the graph of the parent function \( y = \log_2(x) + 5 \) to the right two units and up five units.
    - B. The variables in the table, \( x \) and \( y \), have an inverse variation relationship.
    - C. \( x \)-values increase, \( y \)-values decrease, but the product of their pairs remains constant.

11. How can the relationship between variables in the table be described?
    - A. There is one solution.
    - B. There are two solutions.
    - C. There are no solutions.

12. Use the sequence 100, 95, 90, 85, ... This is an arithmetic sequence in which each term is less than the previous one.
    - a. Describe the sequence in words.
    - b. The next three terms are 80, 75, 70.

13. Water leaks from a 10,000-gal tank at a rate of 5 gal/h. Write a linear model for the situation and use it to find the amount of water in the tank after 24 h.
    - a. \( w = 10,000 - 5t \)
    - b. \( w = 10,000 - 24 \times 5 \)

Extended Response

14. You have a coupon for $10 off a CD, You also get a 20% discount if you show your membership card in the CD club. How much more would you pay if the cashier applies the coupon first?
    - A. $5.00
    - B. $7.00
    - C. $10.00
    - D. $15.00

15. Students work with different size squares. How is the polynomial \( 2x^2 - 3x + 1 \) classified by degree?
    - A. linear
    - B. quadratic
    - C. cubic
    - D. quartic

16. Write a geometric sequence corresponding to these lengths, and a recursive or explicit formula for that sequence.
    - A. \( 3, 6, 12, 24, ... \)
    - B. \( 3, 6, 12, 24, ... \)
    - C. \( 3, 6, 12, 24, ... \)
    - D. \( 3, 6, 12, 24, ... \)
    - E. \( 3, 6, 12, 24, ... \)

17. Student provides incorrect information. No work is shown.
    - A. \( 3, 6, 12, 24, ... \)
    - B. \( 3, 6, 12, 24, ... \)
    - C. \( 3, 6, 12, 24, ... \)
    - D. \( 3, 6, 12, 24, ... \)
    - E. \( 3, 6, 12, 24, ... \)

18. Describe the sequence in words.
    - A. \( 3, 6, 12, 24, ... \)
    - B. \( 3, 6, 12, 24, ... \)
    - C. \( 3, 6, 12, 24, ... \)
    - D. \( 3, 6, 12, 24, ... \)
    - E. \( 3, 6, 12, 24, ... \)

19. The graph of \( f(x) = 2x^2 + 3x - 1 \) and \( g(x) = 2x^2 - 1 \) is a shift of the graph of the parent function \( f(x) = x^2 \), have an inverse variation relationship.
    - A. \( y = x^2 \)
    - B. \( y = x^2 \)
    - C. \( y = x^2 \)
    - D. \( y = x^2 \)
    - E. \( y = x^2 \)

20. How can the relationship between variables in the table be described?
    - A. There is one solution.
    - B. There are two solutions.
    - C. There are no solutions.

21. Describe the sequence in words.
    - A. \( 3, 6, 12, 24, ... \)
    - B. \( 3, 6, 12, 24, ... \)
    - C. \( 3, 6, 12, 24, ... \)
    - D. \( 3, 6, 12, 24, ... \)
    - E. \( 3, 6, 12, 24, ... \)

22. Write a geometric sequence corresponding to these lengths, and a recursive or explicit formula for that sequence.
    - A. \( 3, 6, 12, 24, ... \)
    - B. \( 3, 6, 12, 24, ... \)
    - C. \( 3, 6, 12, 24, ... \)
    - D. \( 3, 6, 12, 24, ... \)
    - E. \( 3, 6, 12, 24, ... \)

23. The variables in the table, \( x \) and \( y \), have an inverse variation relationship.
    - A. \( x \)-values increase, \( y \)-values decrease, but the product of their pairs remains constant.
    - B. \( x \)-values increase, \( y \)-values decrease, but the product of their pairs remains constant.
    - C. \( x \)-values increase, \( y \)-values decrease, but the product of their pairs remains constant.
    - D. \( x \)-values increase, \( y \)-values decrease, but the product of their pairs remains constant.
    - E. \( x \)-values increase, \( y \)-values decrease, but the product of their pairs remains constant.
Activity 3: Analysing
Photographs are often cropped so that only part of the photograph remains. Then, this cropped portion can be reduced or enlarged. Choose a photograph in a textbook. Place a piece of paper over the photograph, trace its original size, and draw a rectangle to indicate a portion of the photograph that you would like to crop. Draw a diagonal from the lower left corner to the upper right corner of the rectangular cropped area. If this diagonal is extended through the upper right corner of the cropped area, and a point selected anywhere along the diagonal or its extension, then the rectangle having the chosen point as its upper right corner (and the same lower left corner as the original cropped area) will have dimensions that are proportional to the dimensions of the cropped area.

- Measure the dimensions and the length of the diagonal of the cropped area.
- Write the first four terms of an arithmetic sequence that has the length of the diagonal of the cropped area as its first term. Using the terms of your sequence as diagonal lengths, find the four corresponding photo widths. What do you notice about this list of widths?
- Write the first four terms of an geometric sequence that has the length of the diagonal of the cropped area as its first term. Using the terms of your sequence as diagonal lengths, find the four corresponding photo widths. What do you notice about this list of widths?

Finishing the Project
The answers to the activities should help you complete your project. Prepare a presentation or demonstration that summarizes how an artist, a designer, or a photographer uses sequences. Present this information to your classmates. Then discuss the sequences you made.

Reflect and Revise
Review your summary. Are your drawings clear and correct? Are your sequences accurate? Practice your presentation in front of at least two people before presenting it to the class. Ask for their suggestions for improvement.

Extending the Project
Geometric and arithmetic patterns are used in other aspects of design and in other careers. Research other areas where sequences are applied.