Model for effective thermal conductivity of nanofluids

Qing-Zhong Xue

Department of Applied Physics, University of Petroleum, Shandong Dongying 257062, PR China

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Abstract

Considering the interface effect between the solid particles and the base fluid in nanofluids, a novel model of the effective thermal conductivity for nanofluids is presented. Based on Maxwell theory and average polarization theory, the formula of calculating the effective thermal conductivity of nanofluids is given. The theoretical results on the effective thermal conductivity of nanotube/oil nanofluid and Al$_2$O$_3$/water nanofluid are in good agreement with the experimental data. Especially, the novel model can interpret the anomalous enhancement of the effective thermal conductivity of nanotube/oil nanofluid and its nonlinearity with nanotube loadings.

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Heating or cooling fluids is important for many industrial sectors, including energy supply and production, transportation and electronics. The thermal conductivity of these fluids plays a vital role in the development of energy-efficient heat transfer equipment [1]. However, conventional heat transfer fluids have poor thermal transfer properties compared to most solids. In order to improve the thermal conductivity of these fluids numerous theoretical and experimental studies of the effective thermal conductivity of liquids containing suspended solid particles have been conducted. Most of the previous studies of the thermal conductivity of suspensions have been confined to those containing milli-sized or micro-sized particles.

Recently, people have demonstrated that nanofluids consisting of CuO or Al$_2$O$_3$ nanoparticles in water or ethylene glycol exhibit enhanced thermal conductivity [2]. A maximum increase in thermal conductivity of approximately 20% was observed in that study for 4 vol% CuO nanoparticles with average diameter 35 nm dispersed in ethylene glycol. A similar behavior has been observed in Al$_2$O$_3$/ethylene glycol nanofluid recently [3]. Furthermore, the effective thermal conductivity has shown to be increased by up to 40% for the nanofluid consisting of ethylene glycol containing approximately 0.3 vol% Cu nanoparticles of mean diameter < 10 nm [1]. And the effective thermal conductivity of nanofluid consisting of carbon nanotube (1 vol%) in oil exhibit 160% enhancement [4].
Table 1
Conventional models of effective thermal conductivity of solid/liquid suspensions. Effective thermal conductivity of solid/liquid suspensions $k_e$, thermal conductivity of base fluid $k_m$, thermal conductivity of particle $k_2$, thermal conductivity ratio $\alpha = k_2/k_m$, $\beta = (\alpha - 1)/(\alpha + 2)$, particle shape factor $n$ and particle volume fraction $\nu$

<table>
<thead>
<tr>
<th>Models</th>
<th>Expressions</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maxwell [9]</td>
<td>$k_{\text{eff}} = 1 + \frac{3(\alpha - 1)\nu}{2(\alpha + 2) - (\alpha - 1)n}$</td>
<td>Spherical particles are considered</td>
</tr>
<tr>
<td>Hamilton–Crosser [10]</td>
<td>$k_{\text{eff}} = \frac{a + (\alpha - 1)(1 + \alpha / 3)}{1 + (\alpha - 1)(1 - \alpha / 3)}$</td>
<td>Spherical and nonspherical particles are considered: $n = 3$ for spheres, $n = 6$ for cylinders</td>
</tr>
<tr>
<td>Jeffrey [11]</td>
<td>$k_{\text{eff}} = 1 + 3\beta \nu + \left(3\beta^2 + \frac{3\nu^2}{4} + \frac{9\nu^3}{16}\right)\nu^2$</td>
<td>High-order terms represent pair interaction of randomly dispersed spheres</td>
</tr>
<tr>
<td>Davis [12]</td>
<td>$k_{\text{eff}} = 1 + \frac{3(\alpha - 1)}{2(\alpha + 2) - (\alpha - 1)n}[\nu + f(\alpha)\nu^2 + O(\nu^3)]$</td>
<td>High-order terms represent pair interaction of randomly dispersed spheres. $f(\alpha) = 2.5$ for $\alpha = 10$, $f(\alpha) = 0.5$ for $\alpha = \infty$</td>
</tr>
<tr>
<td>Lu–Lin [13]</td>
<td>$k_{\text{eff}} = 1 + \alpha \nu + b \nu^2$</td>
<td>Spherical and nonspherical particles are considered</td>
</tr>
<tr>
<td>Bonnecaze–Brady [14,15]</td>
<td>Numerical simulation</td>
<td>Near- and far-field interactions among two or more particles are considered</td>
</tr>
</tbody>
</table>

The experimental study of the thermal conductivity of nanofluids has been intensified in the past few years. However, people find that all the above experimental results are much larger than the theoretical predictions according to the existing models for the effective conductivity of solid/liquid suspension those has been summarized by S.U.S. Chio and coworkers [4] (as shown in Table 1). That is to say, according to the existing model, we cannot understand the effective thermal conductivity of nanofluids. Therefore, we build up a new model to understand the mechanism of the effective thermal conductivity of these nanofluids.

As we know, all the existing theoretical models are only depended on the thermal conductivity of the solid and the liquid and their relative volume fraction, not on the particle size and the interface between the particles and the fluid. In fact, recent experimental study has shown that molecules of normal liquids close to a solid surface organize into layered structures much like a solid [5]. Furthermore, there is evidence that such an organized solid-like structure of a liquid at the surface is a governing factor in heat conduction from a solid wall to an adjacent liquid [6]. Therefore, Choi and the coworkers postulate that this organized solid/liquid interfacial shell makes the transport of energy across the interface effective [4].

Because the interfacial shells are existed between the nanoparticles and the liquid matrix, we can regard both the interfacial shell and the nanoparticle as a “complex nanoparticle”. So, the nanofluid system should be regarded as the complex nanoparticles dispersed in the fluid. Based on this consideration, we build up a new model for calculating the effective thermal conductivity of nanofluids. We assume that $k_e$ is the effective thermal conductivity of the nanofluid; $k_c$ and $k_m$ are the thermal conductivity of the complex nanoparticles and the fluid, respectively.

Let the intensity $\vec{E}$ and heat flux $\vec{q}$ be, respectively, defined as

$$\vec{E} = -\nabla \phi,$$

$$\vec{q} = k \vec{E},$$

where $\phi$ and $k$ are the temperature distribution function and the thermal conductivity, respectively.

Now let us discuss the thermal conductivity of the complex nanoparticles firstly. We assume that the complex nanoparticle is composed of an elliptical nanoparticle of thermal conductivity $k_2$ with half-radii of $(a, b, c)$ and an elliptical shell of thermal conductivity $k_1$ with a thickness of $t$.

The field factor, defined as the ratio between the temperature field $E_c$ in the elliptical nanoparticle to the temperature field $E_s$ in the shell, can be expressed as [7]

$$\beta = \frac{E_c}{E_s} = \sum_{j=x,y,z} \frac{\cos^2 \alpha_j}{1 + B_2j(k_2/k_1 - 1)},$$
where \( \alpha_j \) are the angles between the ellipsoid’s principal axis and the applied field and they satisfy \( \sum_{j=x,y,z} \cos^2 \alpha_j = 1 \), and \( B_{2,j} \) are the depolarization factor component of the elliptical particle along the \( j \)-symmetrical axis. Generally, \( B_{2,j} \) is dependent on the shape and have the form

\[
B_{2,x} = \frac{abc}{2} \int_{0}^{\infty} \frac{du}{(u + a^2)(u + b^2)(u + c^2)},
\]

\( (4) \)

\[
B_{2,y} = \frac{abc}{2} \int_{0}^{\infty} \frac{du}{(u + b^2)(u + a^2)(u + c^2)},
\]

\( (5) \)

\[
B_{2,z} = \frac{abc}{2} \int_{0}^{\infty} \frac{du}{(u + c^2)(u + a^2)(u + b^2)(u + c^2)},
\]

\( (6) \)

\[
B_{2,x} + B_{2,y} + B_{2,z} = 1.
\]

\( (7) \)

We assume that the applied field is along the \( j \)-symmetrical axis. According to Eq. (3), the field factor component along the corresponding symmetrical axis can be expressed as

\[
\beta_j = \frac{E_{c,j}}{E_{s,j}} = \frac{1}{1 + B_{2,j}(k_2/k_1 - 1)}.
\]

\( (8) \)

The effective dielectric constant component \( k_{c,j} \) can be obtained as

\[
k_{c,j} = \frac{\langle q_j \rangle}{\langle E_j \rangle} = \frac{\lambda k_2 E_{c,j} + (1 - \lambda)k_1 E_{s,j}}{\lambda E_{c,j} + (1 - \lambda)E_{s,j}},
\]

\( (9) \)

where \( \lambda = \frac{\text{vol}(	ext{fluid})}{\text{vol}(	ext{fluid}) + \text{vol}(	ext{nanoparticles})} \). \( \langle q_j \rangle \) and \( \langle E_j \rangle \) are the spatial average of the heat flux component and the temperature field component along the axis, respectively.

Substituting Eq. (8) into Eq. (9), we derive the thermal conductivity component along \( j \) axis of the complex elliptical particle

\[
k_{c,j} = k_1 \frac{(1 - B_{2,j})k_1 + B_{2,j}k_2 + (1 - B_{2,j})\lambda(k_2 - k_1)}{(1 - B_{2,j})k_1 + B_{2,j}k_2 - B_{2,j}\lambda(k_2 - k_1)}.
\]

\( (10) \)

We turn now to discuss the thermal conductivity of the whole nanofluid. According to the average theory, we obtain the equation for the effective thermal conductivity of the complex nanoparticle–fluid system (nanofluid) [8]

\[
\sum_{j=x,y,z} \left( 1 - \frac{v}{\lambda} \right) \frac{k_{e,j} - k_{m,j}}{k_{e,j} + B_{m,j}(k_{m,j} - k_{e,j})} + \sum_{j=x,y,z} \frac{v}{\lambda} \frac{k_{e,j} - k_{c,j}}{k_{e,j} + B_{2,j}(k_{c,j} - k_{e,j})} = 0,
\]

\( (11) \)

where \( v \) and \( v/\lambda \) are the volume fraction of the nanoparticles and the complex nanoparticles, respectively.

For simplicity, we assume that all the fluid particles are balls, and all the nanoparticles are the same rotational ellipsoid whose axial length ratio is \( m (m = a/b) \). Based on this assumption and according to Eq. (7), we have

\[
B_{m,j} = 1/3,
\]

\( (12) \)

\[
B_{2,j} = \frac{1 - B_{2,j}}{2}.
\]

\( (13) \)

Substituting Eq. (12) and Eq. (13) into Eq. (11), and considering all the scattering particles have the same spatial probability Eq. (11) reduces to

\[
\left( 1 - \frac{v}{\lambda} \right) \frac{k_e - k_m}{2k_e + k_m} + \frac{v}{\lambda} \frac{k_e - k_{c,x}}{k_e + B_{2,x}(k_{c,x} - k_e)} + \frac{v}{\lambda} \frac{k_e - k_{c,y}}{2k_e + (1 - B_{2,y})(k_{c,y} - k_e)} = 0.
\]

\( (14) \)

Now we apply the novel model to carbon nanotube/oil nanofluid and Al\(_2\)O\(_3\) nanoparticle/water nanofluid. We estimated the thermal conductivity ratios given by the existing models are much smaller than the experimental data and is nonlinear with nanotube volume fraction, whereas those theoretical results given by the existing models are much smaller than the experimental data and linear with nanotube volume fraction (shown in the inset of Fig. 1). For example, the measured enhancement in thermal conductiv-
Fig. 1. Comparison of measured data for carbon nanotube/oil fluid (open circles) and those predicted by the novel model (solid line) and the existing models. The thickness and thermal conductivity of the interfacial shell are fitted as (1) \( t = 1 \text{ nm}; k_1 = 5 \text{ W/mK} \); (2) \( t = 2 \text{ nm}; k_1 = 5 \text{ W/mK} \); (3) \( t = 3 \text{ nm}; k_1 = 5 \text{ W/mK} \); (4) \( t = 3 \text{ nm}; k_1 = 20 \text{ W/mK} \) for the four solid lines, respectively. Because all calculated values given by all the conventional model are almost identical at the low volume fractions, some of the calculated values are shown as dashed lines in the inset where line A = Hamilton–Crosser; line B = Bonnecaze and Brady; line C = Maxwell.

Fig. 2. Comparison of measured data for Al\(_2\)O\(_3\) nanoparticle/water fluid (open circles) and those predicted by the novel model (solid line) and the existing models. The thickness and thermal conductivity of the interfacial shell are fitted as (1) \( t = 1 \text{ nm}; k_1 = 2.1 \text{ W/mK} \); (2) \( t = 3 \text{ nm}; k_1 = 2.1 \text{ W/mK} \); (3) \( t = 5 \text{ nm}; k_1 = 2.1 \text{ W/mK} \); (4) \( t = 5 \text{ nm}; k_1 = 10 \text{ W/mK} \) for the four solid lines, respectively. Because all calculated values given by all the conventional model are almost identical at the low volume fractions, some of the calculated values are shown as the dashed lines in the inset where line A = Bonnecaze and Brady; line B = Maxwell; line C = Davis.

ity for 1 vol% nanotubes in oil is 160%, while the enhancements predicted by the models listed in Table 1 are not more than 10%. In the calculation, the thermal conductivity of nanotube is taken as 2000 W/mK and that of the oil as 0.1448 W/mK [4].

For Al\(_2\)O\(_3\) nanoparticle/water nanofluid the theoretical curve 2 (\( t = 3 \text{ nm}; k_1 = 2.1 \text{ W/mK} \)) given by the novel model is in good agreement with the experimental data and is somewhat nonlinear with the nanoparticle volume fraction, whereas those theoretical results given by the existing models are smaller than the experimental data and linear with the nanoparticle volume fraction (shown in the inset of Fig. 2). In the calculation, the thermal conductivity of Al\(_2\)O\(_3\) nanoparticle is taken as 46 W/mK and that of water as 0.604 W/mK [3].

When the thermal conductivity of the interfacial shell increases the thermal conduction from the nanoparticle to the base fluid get more active, and when the thickness of the interfacial shell gets larger the phonon scattering interface get larger, and furthermore, makes the heat conduction easier. Therefore, as shown in Figs. 1 and 2 we can find that the larger the thickness (or thermal conductivity) of the interfacial shell is, the larger the thermal conductivity of nanofluid is.

In summary, considering the interface effect between the solid particles and the base fluid in nanofluids, a novel model of the effective thermal conductivity for nanofluids is presented. Based on Maxwell theory and average polarization theory, the formula of calculating the effective thermal conductivity of nanofluids is given. The theoretical results on the effective thermal conductivity of nanotube/oil nanofluid and Al\(_2\)O\(_3\)/water nanofluid are in good agreement with the experimental data. Using the novel model we can interpret the anomalous enhancement of the effective thermal conductivity of nanotube/oil nanofluid and its nonlinearity with nanotube loadings. Especially, the novel model can be used to predict the effective thermal conductivity of nanotube in fluid system.
References