Transversals

In the following explanation and drawing, an example of the angles created by two parallel lines and two transversals are shown and explained:

1, 3, 5, 7, 9, 11, 13, 15 are all congruent by vertical angles, corresponding angles, alternate interior angles, and alternate exterior angles 2, 4, 6, 8, 10, 12, 14, 16 are all congruent by vertical angles, corresponding angles, alternate interior angles, and alternate exterior angles 1 and 2 are supplementary 1 and 4 are supplementary 3 and 2 are supplementary 3 and 4 are supplementary 5 and 6 are supplementary 5 and 8 are supplementary 7 and 6 are supplementary 7 and 8 are supplementary 1 and 8 are supplementary 2 and 7 are supplementary 4 and 5 are supplementary 3 and 6 are supplementary And on and on and on...
- Terms

**Acute Angle** - An angle whose measure is less than 90 degrees.

**Adjacent Angle** - Angles that share a vertex, one side, and no interior points.

**Alternate Exterior Angles** - Angles created when a transversal intersects with two lines. Alternate exterior angles lie on opposite sides of the transversal, and on the exterior of the space between the two lines.

**Alternate Interior Angles** - Angles created when a transversal intersects with two lines. Alternate interior angles lie on opposite sides of the transversal, and on the interior of the space between the two lines. That is, they lie between the two lines that intersect with the transversal.

**Angle** - A geometric figure consisting of the union of two rays that share a common endpoint.

**Angle Bisector** - A ray that shares a common vertex with an angle, lies within the interior of that angle, and creates two new angles of equal measure.

**Angle Trisector** - A ray, one of a pair, that shares a common vertex with an angle, lies within the interior of that angle, and creates, with its partner, three new angles of equal
measure. Angle trisectors come in pairs.
Complementary Angles - A pair of angles whose measures sum to 90 degrees. Each angle in the pair is the other's complement.
Congruent - Of the same size. Angles can be congruent to other angles, and segments can be congruent to other segments.
Corresponding Angles - A pair of angles created when a transversal intersects with two lines. Each angle in the pair is on the same side of the transversal, but one is in the exterior of the space created between the lines, and one lies on the interior, between the lines.
Degree - A unit of measure for the size of an angle. One full rotation is equal to 360 degrees. A right angle is 90 degrees. One degree equals $\frac{\pi}{180}$ radians.
Exterior Angle - The larger part of an angle. Were one of the rays of an angle to be rotated until it met the other ray, an exterior angle is spanned by the greater rotation of the two possible rotations. The measure of an exterior angle is always greater than 180 degrees and is always 360 degrees minus the measure of the interior angle that accompanies it. Together, an interior and exterior angle span the entire plane.
Interior Angle - The smaller part of an angle, spanned by the space between the rays that form an angle. Its measure is always less than 180 degrees, and is equal to 360 degrees minus the measure of the exterior angle.
Midpoint - The point on a segment that lies exactly halfway from each end of the segment. The distance from the endpoint of a segment to its midpoint is half the length of the whole segment.
Oblique - Not perpendicular.
Obtuse Angle - An angle whose measure is greater than 90 degrees.
Parallel Lines - Lines that never intersect.
Parallel Postulate - A postulate which states that given a point not located on a line, exactly one line passes through the point that is parallel to original line.

![Figure 9: The parallel postulate](image)

Perpendicular - At a 90 degree angle. A geometric figure (line, segment, plane, etc.) is always perpendicular to another figure.
Perpendicular Bisector - A line or segment that is perpendicular to a segment and contains the midpoint of that segment.
Radian - A unit for measuring the size of an angle. One full rotation is equal to $2\pi$ radians. One radian is equal to $\frac{180}{\pi}$ degrees.
Ray - A portion of a line with a fixed endpoint on one end that extends without bound in the other direction.

Right Angle - A 90 degree angle. It is the angle formed when perpendicular lines or segments intersect.

Segment Bisector - A line or segment that contains the midpoint of a segment.

Straight Angle - A 180 degree angle. Formed by two rays that share a common vertex and point in opposite directions.

Supplementary Angles - A pair of angles whose measures sum to 180 degrees. Each angle in the pair is the other's supplement.

Transversal - A line that intersects with two other lines.

Vertex - The common endpoint of two rays at which an angle is formed.

Vertical Angles - Pairs of angles formed where two lines intersect. These angles are formed by rays pointing in opposite directions, and they are congruent. Vertical angles come in pairs.

Zero Angle - A zero degree angle. It is formed by two rays that share a vertex and point in the same direction.

- Angles

An angle is a geometric figure consisting of two rays with a common endpoint. It looks like this:

\[ \angle ABC \]

Figure %: Angle ABC

The common endpoint is called the vertex of the angle; in this case the vertex is point A, which is a part of the ray AB as well as the ray AC. The angle can be called either angle CAB or angle BAC. The only rule in naming an angle is that the vertex must always be the middle "initial" of the angle. The symbol for an angle is this:

\[ \angle ABC \]

Figure %: The symbol for angle ABC

Measuring Angles

Long ago people wanted to measure angles, so numbers were arbitrarily assigned to determine the size of angles. Under this arbitrary numbering system, one complete
rotation around a point is equal to a 360 degree rotation. (There is another unit of measure for angles besides degrees called radians, in which one full rotation is equal to $2\Pi$ radians; in this text we will use degrees as our default unit for measuring angles.)

**Two angles with the same measure are called congruent angles.** Congruence in angles is symbolized by a small arc drawn in the region between rays. Congruent angles are drawn with the same number of such arcs between their rays. An angle's measure determines how it is classified.

**Zero Angles**

An angle with a measure of zero degrees is called a zero angle. If this is hard to visualize, consider two rays that form some angle greater than zero degrees, like the rays in the . Then picture one of the rays rotating toward the other ray until they both lie in the same line. The angle they create has been shrunk from its original measure to zero degrees. The angle that is now formed has a measure of zero degrees.

![Zero Angle Diagram](image)

*Figure %: A zero angle*

**Right Angles**

An angle with a measure of 90 degrees is called a right angle. A right angle is symbolized with a square drawn in the corner of the angle.

![Right Angle Diagram](image)

*Figure %: A right angle*

**Straight Angles**

An angle with a measure of 180 degrees is called a straight angle. It looks just like a line. Don't mix up straight angles with zero angles.

![Straight Angle Diagram](image)
Acute and Obtuse angles

Another way to classify angles by their measures is to consider whether the angle's measure is greater or less than 90 degrees. If an angle measures less than 90 degrees, it is called an acute angle. If it measures more than 90 degrees, it is called an obtuse angle. Right angles are neither acute nor obtuse. They're just right.

Interior and Exterior Angles

So far, all of the angles we have looked at and studied have been interior angles. When two rays share a common endpoint, two angles are created. Up until now, we have only studied the interior angle: the angle whose measure is less than 180 degrees. But actually, whenever two rays create an angle of less than 180 degrees, they also create another angle whose measure is 360 degrees minus the measure of the smaller angle. As we said before, the smaller angle, whose measure is less than 180 degrees, is the interior angle. The other angle, which seems to rotate around the "outside" of the interior angle, is the exterior angle. The measure of the exterior angle is always greater than that of the interior angle, and is always equal to 360 degrees minus the measure of the interior angle. Below, both are pictured.

Adjacent Angles

In the following sections, we'll study pairs of angles and relationships between angles. In these sections, it will be important to understand properties of angles that lie next to each
other. Formally, these angles are called adjacent angles. Three things must be true for angles to be adjacent:

1. The two angles must share a common vertex.
2. They must share one common side.
3. The angles must not share any interior points.

See how each statement is true for the adjacent angles below.

Angle CAB is adjacent to angle DAB. The angles share a common vertex, A, a common side, ray AB, and share no interior points (the ray AB is not on the interior of either angle, it only forms a side of each).

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**Angle Pairs**

**Complementary and Supplementary Angles**

Special names are given to pairs of angles whose sums equal either 90 or 180 degrees. A pair of angles whose sum is 90 degrees are called complementary angles. Each angle is the other angle's complement. Likewise, if two angles sum to 180 degrees, they are called supplementary angles.

It is important to remember that these terms are only relative. An angle is only supplementary or complementary to another specific angle. A single angle, when considered alone, can be neither supplementary nor complementary--it can only take on one of these properties when considered as one of a pair of angles. Take a look at the examples below.
Vertical Angles

When two lines (or segments) intersect, special names are given to each pair of angles that lie opposite each other. These angles, which are formed by rays that point in opposite directions, are called vertical angles. Vertical angles are always congruent.

![Two pairs of vertical angles.](image)

Angles DAC and BAE are vertical angles. Angles DAB and CAE are also vertical angles.

Parallel Lines

Because lines extend infinitely in both directions, every pair of lines either intersect once, or don't intersect at all. The pairs of lines that never intersect are called parallel lines. Although parallel lines are usually thought of in pairs, an infinite number of lines can be parallel to one another.

![Parallel Lines](image)

The Parallel Postulate

The most important thing to understand about parallel lines is the parallel postulate. It states that through a point not on a line, exactly one line is parallel to that line.

![The parallel postulate](image)
In the above figure, we have line AB and a point C not on the line. The Parallel Postulate states that there exists one line through C which is parallel to line AB. As you know, an infinite number of lines can be drawn through point C, but only one of them will be parallel to line AB.

The parallel postulate is very important in doing geometric proofs. It is basically a way to formally say that when given one line, you can always draw another line somewhere that will be parallel to the given line. In the problem section we'll see how to use the parallel postulate to find the measures of unknown angles.

**Parallel Lines Cut by a Transversal**

Whenever you encounter three lines, and only two of them are parallel, the third line, known as a transversal, will intersect with each of the parallel lines. The angles created by these two intersections have special relationships with one another. See the diagram below.

![Figure %: Parallel lines cut by a transversal](image)

Lines AB and CD are parallel. Line EF, the transversal, is parallel to neither, so it intersects with each. This intersection creates eight angles, numbered one through eight. The special pairs of angles are as follows:

**Corresponding Angles**

Angles, 1 and 5, 2 and 6, 3 and 7, and 4 and 8 are pairs of corresponding angles. Each is on the same side of the transversal as its corresponding angle.

**Alternate Interior Angles**

Angles 4 and 5, and 3 and 6 are pairs of alternate interior angles. They are on opposite sides of the transversal, and between the parallel lines.

**Alternate Exterior Angles**

Angles 1 and 8, and 2 and 7 are pairs of alternate exterior angles. They are on opposite sides of the transversal, and on the exterior of the parallel lines.

**When AB and CD are Parallel**

These eight angles would exist even if lines AB and CD were not parallel. However, when lines AB and CD are parallel, we can draw conclusions about the special angle pairs.

1. The corresponding angles (1 and 5, 2 and 6, etc.) are congruent.
2. The alternate interior angles (4 and 5, 3 and 6) are congruent.
3. The alternate exterior angles (1 and 8, 2 and 7) are congruent.
4. Interior angles on the same side of the transversal (3 and 5, 4 and 6) are supplementary.

These relationships form the bases of many a geometric proof, so it's important to understand them.

- **Perpendicular Lines**

  Lines (or segments) are called perpendicular if their intersection with one another forms a right angle. You can see for yourself that if one of the angles formed by the intersection of two lines or segments is a right angle, then all four angles created will also be right angles.

  ▪ **Perpendicular lines**

  Intersecting lines that are not perpendicular to one another are called oblique lines. Through any given line, there are an infinite number of perpendicular lines. Can you see why?

  ▪ **An infinite number of lines perpendicular to any given line**

  Through a specific point on a line, though, there exists only one perpendicular line. Likewise, given a line and a point not on that line, there is only one perpendicular line through the noncolinear point.

  ▪ **Perpendicular lines through a point on a line, and a point not on that line**
In the picture on the left, line AB contains the point C. There exists only one line, line DE, that contains C and is perpendicular to line AB. In the picture on the right, point C is not on line AB. There exists only one line, line CD, that contains C and is perpendicular to line AB.

**The Distance Between a Line and a Point not on that Line**

When working with geometry it is a common problem to have to find the distance between a line and a point not on that line. There are many different segments that could be drawn between the point and the line, but when you need to find the distance between the point and the line, it is implied that you are seeking the *shortest* distance. This is found by drawing the segment through the point which is perpendicular to the line, and taking its length. The distance between a line and a noncolinear point is represented by this segment.

![Figure](image)

*Figure %: The distance between a line and a point not on that line*

In this figure, the shortest distance between the point C and the line AB is along the segment CD, which is perpendicular to the line.

- **Dividing Angles and Segments**

**Dividing Angles**

Angles can be divided just like ordinary numbers. An angle can only be divided by a ray on the interior of the angle, though. Such a ray that divides an angle into two equal angles is called an angle bisector. Likewise, two rays that divide an angle into three congruent angles are called angle trisectors.

![Figure](image)

*Figure %: An angle bisected and trisected*

On the left, angle ABC is bisected by the ray BD. To know this, we must know that angle ABD and angle CBD are congruent. On the right, angle ABC is trisected by ray BE and ray BF. In this case, the three angles ABE, EBF, and FBC are congruent.
With angle bisectors and trisectors, it also holds true that any of the new angles created by the bisector or trisector is equal to exactly one-half or one-third of the original angle, depending on whether the angle has been bisected or trisected.

**Dividing Segments**

A segment is divided into two equal segments only when a line or segment passes through the midpoint of the original segment. The midpoint of a segment is the point lying in the segment that is exactly halfway from each endpoint of the segment.

![Figure %: The midpoint of a segment](image)

In the above figure, the segment AB is divided into two segments, AM and MB. Point M is the midpoint of segment AB, thus AM and MB are of the same length: one-half the length of AB.

**Bisectors**

When a line or segment passes through the midpoint of another segment, that line or segment is a bisector of the other segment. There are an infinite number of bisectors for every segment, depending on the angle at which the incoming segment or line bisects the other segment.

![Figure %: A segment being bisected by many different lines and segments](image)

The segment AB, with midpoint M, is bisected by segment CD, line EF, and segment GH.

**Perpendicular Bisectors**

If a bisector is perpendicular to the segment it bisects, it is called the perpendicular bisector of that segment. Because there exists only one line perpendicular to a line at a given point, a segment has only one perpendicular bisector: the perpendicular line that passes through the midpoint of the segment.
The line CD contains the midpoint of segment AB, and forms a right angle with the segment. Therefore, it is the perpendicular bisector of segment AB.

Just as there are bisectors for segments, there are trisectors, too. *Segment trisectors* divide a segment into three equal segments.