## Vocabulary Builder

This is an alphabetical list of the key vocabulary terms you will learn in Chapter 4. As you study the chapter, complete each term’s definition or description. Remember to add the page number where you found the term. Add these pages to your Geometry Study Notebook to review vocabulary at the end of the chapter.

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Learning to Read Mathematics

**Proof Builder**

This is a list of key theorems and postulates you will learn in Chapter 4. As you study the chapter, write each theorem or postulate in your own words. Include illustrations as appropriate. Remember to include the page number where you found the theorem or postulate. Add this page to your Geometry Study Notebook so you can review the theorems and postulates at the end of the chapter.

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4-1 Study Guide and Intervention

Classifying Triangles

Classify Triangles by Angles One way to classify a triangle is by the measures of its angles.

- If one of the angles of a triangle is an obtuse angle, then the triangle is an obtuse triangle.
- If one of the angles of a triangle is a right angle, then the triangle is a right triangle.
- If all three of the angles of a triangle are acute angles, then the triangle is an acute triangle.
- If all three angles of an acute triangle are congruent, then the triangle is an equiangular triangle.

Example

Classify each triangle.

a.

All three angles are congruent, so all three angles have measure 60°. The triangle is an equiangular triangle.

b.

The triangle has one angle that is obtuse. It is an obtuse triangle.

c.

The triangle has one right angle. It is a right triangle.

Exercises

Classify each triangle as acute, equiangular, obtuse, or right.

1. 2. 3. 

4. 5. 6.
Classify Triangles by Sides You can classify a triangle by the measures of its sides. Equal numbers of hash marks indicate congruent sides.

- If all three sides of a triangle are congruent, then the triangle is an **equilateral triangle**.
- If at least two sides of a triangle are congruent, then the triangle is an **isosceles triangle**.
- If no two sides of a triangle are congruent, then the triangle is a **scalene triangle**.

### Example

Classify each triangle.

- **a.**
  - Two sides are congruent.
  - The triangle is an isosceles triangle.

- **b.**
  - All three sides are congruent. The triangle is an equilateral triangle.

- **c.**
  - The triangle has no pair of congruent sides. It is a scalene triangle.

### Exercises

Classify each triangle as **equilateral**, **isosceles**, or **scalene**.

1. ![Triangle](image1)
2. ![Triangle](image2)
3. ![Triangle](image3)
4. ![Triangle](image4)
5. ![Triangle](image5)
6. ![Triangle](image6)

7. Find the measure of each side of equilateral \( \triangle RST \) with \( RS = 2x + 2 \), \( ST = 3x \), and \( TR = 5x - 4 \).

8. Find the measure of each side of isosceles \( \triangle ABC \) with \( AB = BC \) if \( AB = 4y \), \( BC = 3y + 2 \), and \( AC = 3y \).

9. Find the measure of each side of \( \triangle ABC \) with vertices \( A(-1, 5), B(6, 1) \), and \( C(2, -6) \). Classify the triangle.
Skills Practice
Classifying Triangles

Use a protractor to classify each triangle as acute, equiangular, obtuse, or right.

1. 

2. 

3. 

4. 

5. 

6. 

Identify the indicated type of triangles.

7. right 

8. isosceles

9. scalene 

10. obtuse

ALGEBRA Find x and the measure of each side of the triangle.

11. \( \triangle ABC \) is equilateral with \( AB = 3x - 2 \), \( BC = 2x + 4 \), and \( CA = x + 10 \).

12. \( \triangle DEF \) is isosceles, \( \angle D \) is the vertex angle, \( DE = x + 7 \), \( DF = 3x - 1 \), and \( EF = 2x + 5 \).

Find the measures of the sides of \( \triangle RST \) and classify each triangle by its sides.

13. \( R(0, 2), S(2, 5), T(4, 2) \)

14. \( R(1, 3), S(4, 7), T(5, 4) \)
Use a protractor to classify each triangle as acute, equiangular, obtuse, or right.

1. 

2. 

3. 

Identify the indicated type of triangles if $\overline{AB} \cong \overline{AD} \cong \overline{BD} \cong \overline{DC}$, $\overline{BE} \cong \overline{ED}$, $\overline{AB} \perp \overline{BC}$, and $\overline{ED} \perp \overline{DC}$.

4. right  
5. obtuse  
6. scalene  
7. isosceles

ALGEBRA Find $x$ and the measure of each side of the triangle.

8. $\triangle FGH$ is equilateral with $FG = x + 5$, $GH = 3x - 9$, and $FH = 2x - 2$.

9. $\triangle LMN$ is isosceles, $\angle L$ is the vertex angle, $LM = 3x - 2$, $LN = 2x + 1$, and $MN = 5x - 2$.

Find the measures of the sides of $\triangle KPL$ and classify each triangle by its sides.

10. $K(-3, 2)$, $P(2, 1)$, $L(-2, -3)$

11. $K(5, -3)$, $P(3, 4)$, $L(-1, 1)$

12. $K(-2, -6)$, $P(-4, 0)$, $L(3, -1)$

13. DESIGN Diana entered the design at the right in a logo contest sponsored by a wildlife environmental group. Use a protractor. How many right angles are there?
Pre-Activity  Why are triangles important in construction?

Read the introduction to Lesson 4-1 at the top of page 178 in your textbook.

• Why are triangles used for braces in construction rather than other shapes?

• Why do you think that isosceles triangles are used more often than scalene triangles in construction?

Reading the Lesson

1. Supply the correct numbers to complete each sentence.
   a. In an obtuse triangle, there are ____ acute angle(s), ____ right angle(s), and ____ obtuse angle(s).
   b. In an acute triangle, there are ____ acute angle(s), ____ right angle(s), and ____ obtuse angle(s).
   c. In a right triangle, there are ____ acute angle(s), ____ right angle(s), and ____ obtuse angle(s).

2. Determine whether each statement is always, sometimes, or never true.
   a. A right triangle is scalene.
   b. An obtuse triangle is isosceles.
   c. An equilateral triangle is a right triangle.
   d. An equilateral triangle is isosceles.
   e. An acute triangle is isosceles.
   f. A scalene triangle is obtuse.

3. Describe each triangle by as many of the following words as apply: acute, obtuse, right, scalene, isosceles, or equilateral.
   a. 
   b. 
   c. 

Helping You Remember

4. A good way to remember a new mathematical term is to relate it to a nonmathematical definition of the same word. How is the use of the word acute, when used to describe acute pain, related to the use of the word acute when used to describe an acute angle or an acute triangle?
Reading Mathematics

When you read geometry, you may need to draw a diagram to make the text easier to understand.

Example

Consider three points, A, B, and C on a coordinate grid. The y-coordinates of A and B are the same. The x-coordinate of B is greater than the x-coordinate of A. Both coordinates of C are greater than the corresponding coordinates of B. Is triangle ABC acute, right, or obtuse?

To answer this question, first draw a sample triangle that fits the description. Side AB must be a horizontal segment because the y-coordinates are the same. Point C must be located to the right and up from point B.

From the diagram you can see that triangle ABC must be obtuse.

Answer each question. Draw a simple triangle on the grid above to help you.

1. Consider three points, R, S, and T on a coordinate grid. The x-coordinates of R and S are the same. The y-coordinate of T is between the y-coordinates of R and S. The x-coordinate of T is less than the x-coordinate of R. Is angle R of triangle RST acute, right, or obtuse?

2. Consider three noncollinear points, J, K, and L on a coordinate grid. The y-coordinates of J and K are the same. The x-coordinates of K and L are the same. Is triangle JKL acute, right, or obtuse?

3. Consider three noncollinear points, D, E, and F on a coordinate grid. The x-coordinates of D and E are opposites. The y-coordinates of D and E are the same. The x-coordinate of F is 0. What kind of triangle must \( \triangle DEF \) be: scalene, isosceles, or equilateral?

4. Consider three points, G, H, and I on a coordinate grid. Points G and H are on the positive y-axis, and the y-coordinate of G is twice the y-coordinate of H. Point I is on the positive x-axis, and the x-coordinate of I is greater than the y-coordinate of G. Is triangle GHI scalene, isosceles, or equilateral?
### Angle Sum Theorem

If the measures of two angles of a triangle are known, the measure of the third angle can always be found.

**Example 1**

Find \( m \angle T \).

\[
R + S + T = 180 \\
25 + 35 + m \angle T = 180 \\
60 + m \angle T = 180 \\
m \angle T = 120
\]

**Example 2**

Find the missing angle measures.

\[
1 + A + B = 180 \\
1 + 58 + 90 = 180 \\
1 + 148 = 180 \\
m \angle 1 = 32 \\
m \angle 2 = 32 \\
m \angle 3 + 32 + 108 = 180 \\
m \angle 3 + 140 = 180 \\
m \angle 3 = 40
\]

### Exercises

Find the measure of each numbered angle.

1. \( \triangle PQR \):
   - \( \angle P = 90^\circ \)
   - \( \angle Q = 62^\circ \)
   - \( \angle R \)?

2. \( \triangle SQR \):
   - \( \angle S = 130^\circ \)
   - \( \angle R \)?

3. \( \triangle WUT \):
   - \( \angle U = 30^\circ \)
   - \( \angle W = 60^\circ \)
   - \( \angle T \)?

4. \( \triangle MNP \):
   - \( \angle M = 66^\circ \)
   - \( \angle N = 58^\circ \)
   - \( \angle P \)?

5. \( \triangle RST \):
   - \( \angle R = 60^\circ \)
   - \( \angle S = 30^\circ \)
   - \( \angle T \)?
Exterior Angle Theorem At each vertex of a triangle, the angle formed by one side and an extension of the other side is called an exterior angle of the triangle. For each exterior angle of a triangle, the remote interior angles are the interior angles that are not adjacent to that exterior angle. In the diagram below, \( \angle B \) and \( \angle A \) are the remote interior angles for exterior \( \angle DCB \).

Exterior Angle Theorem

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<th>The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles. ( m\angle 1 = m\angle A + m\angle B )</th>
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**Example 1**

Find \( m\angle 1 \).

\[
\begin{align*}
m\angle 1 &= m\angle R + m\angle S \\
&= 60^\circ + 80^\circ \\
&= 140^\circ
\end{align*}
\]

**Example 2**

Find \( x \).

\[
\begin{align*}
m\angle PQS &= m\angle R + m\angle S \\
78^\circ &= 55^\circ + x \\
23^\circ &= x
\end{align*}
\]

**Exercises**

Find the measure of each numbered angle.

1. \( \angle X \)

2. \( \angle A \)

3. \( \angle N \)

4. \( \angle R \)

Find \( x \).

5. \( \angle A \)

6. \( \angle E \)
Skills Practice
Angles of Triangles

Find the missing angle measures.
1. 2.

Find the measure of each angle.
3. \(\angle 1\)
4. \(\angle 2\)
5. \(\angle 3\)

Find the measure of each angle.
6. \(\angle 1\)
7. \(\angle 2\)
8. \(\angle 3\)

Find the measure of each angle.
9. \(\angle 1\)
10. \(\angle 2\)
11. \(\angle 3\)
12. \(\angle 4\)
13. \(\angle 5\)

Find the measure of each angle.
14. \(\angle 1\)
15. \(\angle 2\)
4-2 Practice
Angles of Triangles

Find the missing angle measures.

1. [Diagram]

2. [Diagram with angles 40° and 55°]

Find the measure of each angle.

3. \(m\angle 1\)

4. \(m\angle 2\)

5. \(m\angle 3\)

Find the measure of each angle.

6. \(m\angle 1\)

7. \(m\angle 4\)

8. \(m\angle 3\)

9. \(m\angle 2\)

10. \(m\angle 5\)

11. \(m\angle 6\)

Find the measure of each angle if \(\angle BAD\) and \(\angle BDC\) are right angles and \(m\angle ABC = 84\).

12. \(m\angle 1\)

13. \(m\angle 2\)

14. **CONSTRUCTION** The diagram shows an example of the Pratt Truss used in bridge construction. Use the diagram to find \(m\angle 1\).
Pre-Activity  How are the angles of triangles used to make kites?

Read the introduction to Lesson 4-2 at the top of page 185 in your textbook.

The frame of the simplest kind of kite divides the kite into four triangles. Describe these four triangles and how they are related to each other.

Reading the Lesson

1. Refer to the figure.
   a. Name the three interior angles of the triangle. (Use three letters to name each angle.)
   b. Name three exterior angles of the triangle. (Use three letters to name each angle.)
   c. Name the remote interior angles of \( \angle EAB \).
   d. Find the measure of each angle without using a protractor.
      i. \( \angle DBC \)  
      ii. \( \angle ABC \)  
      iii. \( \angle ACF \)  
      iv. \( \angle EAB \)

2. Indicate whether each statement is true or false. If the statement is false, replace the underlined word or number with a word or number that will make the statement true.
   a. The acute angles of a right triangle are \underline{supplementary}.
   b. The sum of the measures of the angles of any triangle is \underline{100}.
   c. A triangle can have at most one right angle or \underline{acute} angle.
   d. If two angles of one triangle are congruent to two angles of another triangle, then the third angles of the triangles are \underline{congruent}.
   e. The measure of an exterior angle of a triangle is equal to the \underline{difference} of the measures of the two remote interior angles.
   f. If the measures of two angles of a triangle are 62 and 93, then the measure of the third angle is \underline{35}.
   g. An \underline{exterior} angle of a triangle forms a linear pair with an interior angle of the triangle.

Helping You Remember

3. Many students remember mathematical ideas and facts more easily if they see them demonstrated visually rather than having them stated in words. Describe a visual way to demonstrate the Angle Sum Theorem.
Finding Angle Measures in Triangles

You can use algebra to solve problems involving triangles.

Example

In triangle $ABC$, $m\angle A$ is twice $m\angle B$, and $m\angle C$ is 8 more than $m\angle B$. What is the measure of each angle?

Write and solve an equation. Let $x = m\angle B$.

$$m\angle A + m\angle B + m\angle C = 180$$

$$2x + x + (x + 8) = 180$$

$$4x + 8 = 180$$

$$4x = 172$$

$$x = 43$$

So, $m\angle A = 2(43)$ or 86, $m\angle B = 43$, and $m\angle C = 43 + 8$ or 51.

Solve each problem.

1. In triangle $DEF$, $m\angle E$ is three times $m\angle D$, and $m\angle F$ is 9 less than $m\angle E$. What is the measure of each angle?

2. In triangle $RST$, $m\angle T$ is 5 more than $m\angle R$, and $m\angle S$ is 10 less than $m\angle T$. What is the measure of each angle?

3. In triangle $JKL$, $m\angle K$ is four times $m\angle J$, and $m\angle L$ is five times $m\angle J$. What is the measure of each angle?

4. In triangle $XYZ$, $m\angle Z$ is 2 more than twice $m\angle X$, and $m\angle Y$ is 7 less than twice $m\angle X$. What is the measure of each angle?

5. In triangle $GHI$, $m\angle H$ is 20 more than $m\angle G$, and $m\angle G$ is 8 more than $m\angle I$. What is the measure of each angle?

6. In triangle $MNO$, $m\angle M$ is equal to $m\angle N$, and $m\angle O$ is 5 more than three times $m\angle N$. What is the measure of each angle?

7. In triangle $STU$, $m\angle U$ is half $m\angle T$, and $m\angle S$ is 30 more than $m\angle T$. What is the measure of each angle?

8. In triangle $PQR$, $m\angle P$ is equal to $m\angle Q$, and $m\angle R$ is 24 less than $m\angle P$. What is the measure of each angle?

9. Write your own problems about measures of triangles.
4-3 Study Guide and Intervention

**Congruent Triangles**

**Corresponding Parts of Congruent Triangles**

Triangles that have the same size and same shape are **congruent triangles**. Two triangles are congruent if and only if all three pairs of corresponding angles are congruent and all three pairs of corresponding sides are congruent. In the figure, \( \triangle ABC \cong \triangle RST \).

**Example**

If \( \triangle XYZ \cong \triangle RST \), name the pairs of congruent angles and congruent sides.

\[
\angle X \cong \angle R, \quad \angle Y \cong \angle S, \quad \angle Z \cong \angle T \\
XY \cong RS, \quad XZ \cong RT, \quad YZ \cong ST
\]

**Exercises**

Identify the congruent triangles in each figure.

1.  
2.  
3.  

Name the corresponding congruent angles and sides for the congruent triangles.

4.  
5.  
6.
Identify Congruence Transformations

If two triangles are congruent, you can slide, flip, or turn one of the triangles and they will still be congruent. These are called congruence transformations because they do not change the size or shape of the figure. It is common to use prime symbols to distinguish between an original \( \triangle ABC \) and a transformed \( \triangle A'B'C' \).

**Example**

Name the congruence transformation that produces \( \triangle A'B'C' \) from \( \triangle ABC \).

The congruence transformation is a slide.

\[ \angle A \cong \angle A'; \angle B \cong \angle B'; \angle C \cong \angle C'; \]
\[ AB \cong A'B'; AC \cong A'C'; BC \cong B'C' \]

**Exercises**

Describe the congruence transformation between the two triangles as a *slide*, a *flip*, or a *turn*. Then name the congruent triangles.

1. 
2. 
3. 
4. 
5. 
6.
Skills Practice

Congruent Triangles

Identify the congruent triangles in each figure.

1. 

2. 

3. 

4. 

Name the congruent angles and sides for each pair of congruent triangles.

5. \( \triangle ABC \cong \triangle FGH \)

6. \( \triangle PQR \cong \triangle STU \)

Verify that each of the following transformations preserves congruence, and name the congruence transformation.

7. \( \triangle ABC \cong \triangle A'B'C' \)

8. \( \triangle DEF \cong \triangle D'E'F' \)
4-3 Practice

Congruent Triangles

Identify the congruent triangles in each figure.

1. 

2. 

Name the congruent angles and sides for each pair of congruent triangles.

3. \( \triangle GKP \cong \triangle L MN \)

4. \( \triangle ANC \cong \triangle RBV \)

Verify that each of the following transformations preserves congruence, and name the congruence transformation.

5. \( \triangle PST \cong \triangle P'S'T' \)

6. \( \triangle LMN \cong \triangle L'M'N' \)

QUILTING For Exercises 7 and 8, refer to the quilt design.

7. Indicate the triangles that appear to be congruent.

8. Name the congruent angles and congruent sides of a pair of congruent triangles.
4-3  Reading to Learn Mathematics

Congruent Triangles

Pre-Activity  Why are triangles used in bridges?

Read the introduction to Lesson 4-3 at the top of page 192 in your textbook.

In the bridge shown in the photograph in your textbook, diagonal braces were used to divide squares into two isosceles right triangles. Why do you think these braces are used on the bridge?

Reading the Lesson

1. If \( \triangle RST \equiv \triangle UWV \), complete each pair of congruent parts.

\[
\angle R \equiv \underline{\quad} \quad \underline{\quad} \equiv \angle W \quad \angle T \equiv \underline{\quad}
\]

\[
\overline{RT} \equiv \underline{\quad} \quad \underline{\quad} \equiv \overline{UW} \quad \underline{\quad} \equiv \overline{WV}
\]

2. Identify the congruent triangles in each diagram.

   a.  
   b.  
   c.  
   d.  

3. Determine whether each statement says that congruence of triangles is reflexive, symmetric, or transitive.

   a. If the first of two triangles is congruent to the second triangle, then the second triangle is congruent to the first.

   b. If there are three triangles for which the first is congruent to the second and the second is congruent to the third, then the first triangle is congruent to the third.

   c. Every triangle is congruent to itself.

Helping You Remember

4. A good way to remember something is to explain it to someone else. Your classmate Ben is having trouble writing congruence statements for triangles because he thinks he has to match up three pairs of sides and three pairs of angles. How can you help him understand how to write correct congruence statements more easily?
Transformations in The Coordinate Plane

The following statement tells one way to map preimage points to image points in the coordinate plane.

\[(x, y) \rightarrow (x + 6, y - 3)\]

This can be read, “The point with coordinates \((x, y)\) is mapped to the point with coordinates \((x + 6, y - 3)\).” With this transformation, for example, \((3, 5)\) is mapped to \((3 + 6, 5 - 3)\) or \((9, 2)\). The figure shows how the triangle \(ABC\) is mapped to triangle \(XYZ\).

1. Does the transformation above appear to be a congruence transformation? Explain your answer.

Draw the transformation image for each figure. Then tell whether the transformation is or is not a congruence transformation.

2. \((x, y) \rightarrow (x - 4, y)\)

3. \((x, y) \rightarrow (x + 8, y + 7)\)

4. \((x, y) \rightarrow (-x, -y)\)

5. \((x, y) \rightarrow \left(-\frac{1}{2}x, y\right)\)
SSS Postulate  You know that two triangles are congruent if corresponding sides are congruent and corresponding angles are congruent. The Side-Side-Side (SSS) Postulate lets you show that two triangles are congruent if you know only that the sides of one triangle are congruent to the sides of the second triangle.

| SSS Postulate | If the sides of one triangle are congruent to the sides of a second triangle, then the triangles are congruent. |

**Example**  Write a two-column proof.

Given: \( AB \cong DB \) and \( C \) is the midpoint of \( AD \).

Prove: \( \triangle ABC \cong \triangle DBC \)

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<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
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</thead>
<tbody>
<tr>
<td>1. ( AB \cong DB )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( C ) is the midpoint of ( AD ).</td>
<td>2. Given</td>
</tr>
<tr>
<td>3. ( AC \cong DC )</td>
<td>3. Definition of midpoint</td>
</tr>
<tr>
<td>4. ( BC \cong BC )</td>
<td>4. Reflexive Property of ( \cong )</td>
</tr>
<tr>
<td>5. ( \triangle ABC \cong \triangle DBC )</td>
<td>5. SSS Postulate</td>
</tr>
</tbody>
</table>

**Exercises**

Write a two-column proof.

1. \( \triangle ABC \cong \triangle XYZ \)

   Given: \( AB \cong XY, AC \cong XZ, BC \cong YZ \)

   Prove: \( \triangle ABC \cong \triangle XYZ \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
</table>

2. \( \triangle RST \cong \triangle UTS \)

   Given: \( RS \cong UT, RT \cong US \)

   Prove: \( \triangle RST \cong \triangle UTS \)

| Statements | Reasons |
Example

For each diagram, determine which pairs of triangles can be proved congruent by the SAS Postulate.

a. In $\triangle ABC$, the angle is not “included” by the sides $AB$ and $AC$. So the triangles cannot be proved congruent by the SAS Postulate.

b. The right angles are congruent and they are the included angles for the congruent sides. $\triangle DEF \cong \triangle JGH$ by the SAS Postulate.

c. The included angles, $\angle 1$ and $\angle 2$, are congruent because they are alternate interior angles for two parallel lines. $\triangle PSR \cong \triangle RQP$ by the SAS Postulate.

Exercises

For each figure, determine which pairs of triangles can be proved congruent by the SAS Postulate.

1. $\triangle NPR$ and $\triangle NUR$

2. $\triangle XYZ$ and $\triangle WYZ$

3. $\triangle NPL$ and $\triangle MNL$

4. $\triangle MVT$ and $\triangle MTV$

5. $\triangle ABC$ and $\triangle BCD$

6. $\triangle JHG$ and $\triangle HJG$
Skills Practice

Proving Congruence—SSS, SAS

Determine whether \( \triangle ABC \cong \triangle KLM \) given the coordinates of the vertices. Explain.

1. \( A(-3, 3), B(-1, 3), C(-3, 1), K(1, 4), L(3, 4), M(1, 6) \)

2. \( A(-4, -2), B(-4, 1), C(-1, -1), K(0, -2), L(0, 1), M(4, 1) \)

3. Write a flow proof.
   
   Given: \( \overline{PR} \cong \overline{DE}, \overline{PT} \cong \overline{DF} \)
   \( \angle R \cong \angle E, \angle T \cong \angle F \)
   
   Prove: \( \triangle PRT \cong \triangle DEF \)

Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove that they are congruent, write *not possible*.

4.  

5.  

6.  

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Glencoe Geometry
4-4 Practice

Proving Congruence—SSS, SAS

Determine whether $\triangle DEF \cong \triangle PQR$ given the coordinates of the vertices. Explain.

1. $D(-6, 1), E(1, 2), F(-1, -4), P(0, 5), Q(7, 6), R(5, 0)$

2. $D(-7, -3), E(-4, -1), F(-2, -5), P(2, -2), Q(5, -4), R(0, -5)$

3. Write a flow proof.
   Given: $RS = TS$
   $V$ is the midpoint of $RT$.
   Prove: $\triangle RSV \cong \triangle TSV$

Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove that they are congruent, write *not possible*.

4. 

5. 

6. 

7. INDIRECT MEASUREMENT To measure the width of a sinkhole on his property, Harmon marked off congruent triangles as shown in the diagram. How does he know that the lengths $A'B'$ and $AB$ are equal?
**Pre-Activity**

How do land surveyors use congruent triangles?

Read the introduction to Lesson 4-4 at the top of page 200 in your textbook.

Why do you think that land surveyors would use congruent right triangles rather than other congruent triangles to establish property boundaries?

**Reading the Lesson**

1. Refer to the figure.
   a. Name the sides of \( \triangle LMN \) for which \( \angle L \) is the included angle.
   b. Name the sides of \( \triangle LMN \) for which \( \angle N \) is the included angle.
   c. Name the sides of \( \triangle LMN \) for which \( \angle M \) is the included angle.

2. Determine whether you have enough information to prove that the two triangles in each figure are congruent. If so, write a congruence statement and name the congruence postulate that you would use. If not, write not possible.
   a.
   b.
   c. \( EH \) and \( DG \) bisect each other.
   d.

**Helping You Remember**

3. Find three words that explain what it means to say that two triangles are congruent and that can help you recall the meaning of the SSS Postulate.
Congruent Parts of Regular Polygonal Regions

Congruent figures are figures that have exactly the same size and shape. There are many ways to divide regular polygonal regions into congruent parts. Three ways to divide an equilateral triangular region are shown. You can verify that the parts are congruent by tracing one part, then rotating, sliding, or reflecting that part on top of the other parts.

1. Divide each square into four congruent parts. Use three different ways.

2. Divide each pentagon into five congruent parts. Use three different ways.

3. Divide each hexagon into six congruent parts. Use three different ways.

4. What hints might you give another student who is trying to divide figures like those into congruent parts?
ASA Postulate  The Angle-Side-Angle (ASA) Postulate lets you show that two triangles are congruent.

ASA Postulate  If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.

Example  Find the missing congruent parts so that the triangles can be proved congruent by the ASA Postulate. Then write the triangle congruence.

a. 

Two pairs of corresponding angles are congruent, \( \angle A \cong \angle D \) and \( \angle C \cong \angle F \). If the included sides \( \overline{AC} \) and \( \overline{DF} \) are congruent, then \( \triangle ABC \cong \triangle DEF \) by the ASA Postulate.

b. 

\( \angle R \cong \angle Y \) and \( \overline{SR} \cong \overline{XY} \). If \( \angle S \cong \angle X \), then \( \triangle RST \cong \triangle YXW \) by the ASA Postulate.

Exercises  What corresponding parts must be congruent in order to prove that the triangles are congruent by the ASA Postulate? Write the triangle congruence statement.

1. 

2. 

3. 

4. 

5. 

6.
AAS Theorem  Another way to show that two triangles are congruent is the Angle-Angle-Side (AAS) Theorem.

| AAS Theorem | If two angles and a nonincluded side of one triangle are congruent to the corresponding two angles and side of a second triangle, then the two triangles are congruent. |

You now have five ways to show that two triangles are congruent.

• definition of triangle congruence
• ASA Postulate
• SSS Postulate
• AAS Theorem
• SAS Postulate

Example  In the diagram, \( \angle BCA \cong \angle DCA \). Which sides are congruent? Which additional pair of corresponding parts needs to be congruent for the triangles to be congruent by the AAS Postulate?

\( \overline{AC} \cong \overline{AC} \) by the Reflexive Property of congruence. The congruent angles cannot be \( \angle 1 \) and \( \angle 2 \), because \( \overline{AC} \) would be the included side. If \( \angle B \cong \angle D \), then \( \triangle ABC \cong \triangle ADC \) by the AAS Theorem.

Exercises  In Exercises 1 and 2, draw and label \( \triangle ABC \) and \( \triangle DEF \). Indicate which additional pair of corresponding parts needs to be congruent for the triangles to be congruent by the AAS Theorem.

1. \( \angle A \cong \angle D; \angle B \cong \angle E \)  
2. \( \overline{BC} \cong \overline{EF}; \angle A \cong \angle D \)

3. Write a flow proof.

Given: \( \angle S \cong \angle U; \overline{TR} \) bisects \( \angle STU \).

Prove: \( \angle SRT \cong \angle URT \)
Write a flow proof.

1. Given: \( \angle N \cong \angle L \)
   \( JK \cong MK \)
   Prove: \( \triangle JKN \cong \triangle MKL \)

2. Given: \( AB \cong CB \)
   \( \angle A \cong \angle C \)
   \( DB \) bisects \( \angle ABC \).
   Prove: \( AD \cong CD \)

3. Write a paragraph proof.
   Given: \( DE \parallel FG \)
   \( \angle E \cong \angle G \)
   Prove: \( \triangle DFG \cong \triangle FDE \)
1. Write a flow proof.
   **Given:** \( S \) is the midpoint of \( QT \).  
   \( QR \parallel TU \)
   **Prove:** \( \triangle QSR \cong \triangle TSU \)

2. Write a paragraph proof.
   **Given:** \( \angle D \cong \angle F \)
   \( GE \) bisects \( \angle DEF \).
   **Prove:** \( DG \cong FG \)

ARCHITECTURE  For Exercises 3 and 4, use the following information.
An architect used the window design in the diagram when remodeling an art studio. \( AB \) and \( CB \) each measure 3 feet.

3. Suppose \( D \) is the midpoint of \( AC \). Determine whether \( \triangle ABD \cong \triangle CBD \).
   Justify your answer.

4. Suppose \( \angle A \cong \angle C \). Determine whether \( \triangle ABD \cong \triangle CBD \). Justify your answer.
Pre-Activity  How are congruent triangles used in construction?

Read the introduction to Lesson 4-5 at the top of page 207 in your textbook. Which of the triangles in the photograph in your textbook appear to be congruent?

Reading the Lesson

1. Explain in your own words the difference between how the ASA Postulate and the AAS Theorem are used to prove that two triangles are congruent.

2. Which of the following conditions are sufficient to prove that two triangles are congruent?
   A. Two sides of one triangle are congruent to two sides of the other triangle.
   B. The three sides of one triangles are congruent to the three sides of the other triangle.
   C. The three angles of one triangle are congruent to the three angles of the other triangle.
   D. All six corresponding parts of two triangles are congruent.
   E. Two angles and the included side of one triangle are congruent to two sides and the included angle of the other triangle.
   F. Two sides and a nonincluded angle of one triangle are congruent to two sides and a nonincluded angle of the other triangle.
   G. Two angles and a nonincluded side of one triangle are congruent to two angles and the corresponding nonincluded side of the other triangle.
   H. Two sides and the included angle of one triangle are congruent to two sides and the included angle of the other triangle.
   I. Two angles and a nonincluded side of one triangle are congruent to two angles and a nonincluded side of the other triangle.

3. Determine whether you have enough information to prove that the two triangles in each figure are congruent. If so, write a congruence statement and name the congruence postulate or theorem that you would use. If not, write not possible.
   a.  
   b.  $T$ is the midpoint of $RU$.

Helping You Remember

4. A good way to remember mathematical ideas is to summarize them in a general statement. If you want to prove triangles congruent by using three pairs of corresponding parts, what is a good way to remember which combinations of parts will work?
Congruent Triangles in the Coordinate Plane

If you know the coordinates of the vertices of two triangles in the coordinate plane, you can often decide whether the two triangles are congruent. There may be more than one way to do this.

1. Consider \( \triangle ABD \) and \( \triangle CDB \) whose vertices have coordinates \( A(0, 0) \), \( B(2, 5) \), \( C(9, 5) \), and \( D(7, 0) \). Briefly describe how you can use what you know about congruent triangles and the coordinate plane to show that \( \triangle ABD \cong \triangle CDB \). You may wish to make a sketch to help get you started.

2. Consider \( \triangle PQR \) and \( \triangle KLM \) whose vertices are the following points.
   
   \[ P(1, 2) \quad Q(3, 6) \quad R(6, 5) \]
   
   \[ K(-2, 1) \quad L(-6, 3) \quad M(-5, 6) \]
   
   Briefly describe how you can show that \( \triangle PQR \cong \triangle KLM \).

3. If you know the coordinates of all the vertices of two triangles, is it always possible to tell whether the triangles are congruent? Explain.
Properties of Isosceles Triangles  An isosceles triangle has two congruent sides. The angle formed by these sides is called the vertex angle. The other two angles are called base angles. You can prove a theorem and its converse about isosceles triangles.

- If two sides of a triangle are congruent, then the angles opposite those sides are congruent. (Isosceles Triangle Theorem)
- If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

Example 1

Find \( x \).

\[
\begin{align*}
BC &= BA, \text{ so} \\
m\angle A &= m\angle C. \\
5x - 10 &= 4x + 5 \\
x - 10 &= 5 \\
x &= 15
\end{align*}
\]

Example 2

Find \( x \).

\[
\begin{align*}
m\angle S &= m\angle T, \text{ so} \\
SR &= TR. \\
3x - 13 &= 2x \\
3x &= 2x + 13 \\
x &= 13
\end{align*}
\]

Exercises

Find \( x \).

1. \[
\begin{align*}
\angle P &= 40^\circ, \quad \angle Q = 2x^\circ \\
\angle R &= 2x^\circ \\
\end{align*}
\]

2. \[
\begin{align*}
\angle S &= 2x + 6 \\
\angle T &= 3x - 6
\end{align*}
\]

3. \[
\begin{align*}
\angle W &= 3x^\circ \\
\angle Y &= 3x^\circ \\
\angle Z &= 3x^\circ
\end{align*}
\]

4. \[
\begin{align*}
\angle T &= (6x + 6)^\circ \\
\angle P &= (6x + 6)^\circ \\
\angle D &= 2x^\circ
\end{align*}
\]

5. \[
\begin{align*}
\angle G &= 30^\circ \\
\angle B &= 3x^\circ \\
\angle L &= 3x^\circ
\end{align*}
\]

6. \[
\begin{align*}
\angle R &= 3x^\circ \\
\angle T &= 3x^\circ \\
\angle S &= 3x^\circ
\end{align*}
\]

7. Write a two-column proof.

Given: \( \angle 1 \equiv \angle 2 \)

Prove: \( AB \equiv CB \)

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</table>
Properties of Equilateral Triangles  An equilateral triangle has three congruent sides. The Isosceles Triangle Theorem can be used to prove two properties of equilateral triangles.

1. A triangle is equilateral if and only if it is equiangular.
2. Each angle of an equilateral triangle measures 60°.

Example  Prove that if a line is parallel to one side of an equilateral triangle, then it forms another equilateral triangle.

Proof:

<table>
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<th>Statements</th>
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<tbody>
<tr>
<td>1. △ABC is equilateral; PQ ∥ BC.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. m∠A = m∠B = m∠C = 60°</td>
<td>2. Each ∠ of an equilateral △ measures 60°.</td>
</tr>
<tr>
<td>3. ∠1 ≅ ∠B, ∠2 ≅ ∠C</td>
<td>3. If</td>
</tr>
<tr>
<td>4. m∠1 = 60°, m∠2 = 60°</td>
<td>4. Substitution</td>
</tr>
<tr>
<td>5. △APQ is equilateral.</td>
<td>5. If a △ is equiangular, then it is equilateral.</td>
</tr>
</tbody>
</table>

Exercises

Find x.

1. 2. 3.

4. 5. 6.

7. Write a two-column proof.

   Given: △ABC is equilateral; ∠1 ≅ ∠2.
   Prove: ∠ADB ≅ ∠CDB

Proof:

<table>
<thead>
<tr>
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</thead>
</table>
Skills Practice

Isosceles Triangles

Refer to the figure.

1. If $\overline{AC} \cong \overline{AD}$, name two congruent angles.

2. If $\overline{BE} \cong \overline{BC}$, name two congruent angles.

3. If $\angle EBA \cong \angle EAB$, name two congruent segments.

4. If $\angle CED \cong \angle CDE$, name two congruent segments.

$\triangle ABF$ is isosceles, $\triangle CDF$ is equilateral, and $m\angle AFD = 150$. Find each measure.

5. $m\angle CFD$
6. $m\angle AFB$
7. $m\angle ABF$
8. $m\angle A$

In the figure, $\overline{PL} \cong \overline{RL}$ and $\overline{LR} \cong \overline{BL}$.

9. If $m\angle RLP = 100$, find $m\angle BRL$.
10. If $m\angle LPR = 34$, find $m\angle B$.

11. Write a two-column proof.

   **Given:** $\overline{CD} \cong \overline{CG}$
   $\overline{DE} \cong \overline{GF}$

   **Prove:** $\overline{CE} \cong \overline{CF}$
4-6 Practice

Isosceles Triangles

Refer to the figure.

1. If $\overline{RV} \cong \overline{RT}$, name two congruent angles.

2. If $\overline{RS} \cong \overline{SV}$, name two congruent angles.

3. If $\angle SRT \cong \angle STR$, name two congruent segments.

4. If $\angle STV \cong \angle SVT$, name two congruent segments.

Triangles $GHM$ and $HJM$ are isosceles, with $\overline{GH} \cong \overline{MH}$ and $\overline{HJ} \cong \overline{MJ}$. Triangle $KLM$ is equilateral, and $m\angle HMK = 50$. Find each measure.

5. $m\angle KML$  
6. $m\angle HMG$  
7. $m\angle GHM$

8. If $m\angle HJM = 145$, find $m\angle MHJ$.

9. If $m\angle G = 67$, find $m\angle GHM$.

10. Write a two-column proof.
    
    Given: $\overline{DE} \parallel \overline{BC}$
    
    $\angle 1 \cong \angle 2$
    
    Prove: $AB \cong AC$

11. SPORTS A pennant for the sports teams at Lincoln High School is in the shape of an isosceles triangle. If the measure of the vertex angle is 18, find the measure of each base angle.
Pre-Activity  How are triangles used in art?

Read the introduction to Lesson 4-6 at the top of page 216 in your textbook.

• Why do you think that isosceles and equilateral triangles are used more often than scalene triangles in art?

• Why might isosceles right triangles be used in art?

Reading the Lesson

1. Refer to the figure.
   a. What kind of triangle is $\triangle QRS$?
   b. Name the legs of $\triangle QRS$.
   c. Name the base of $\triangle QRS$.
   d. Name the vertex angle of $\triangle QRS$.
   e. Name the base angles of $\triangle QRS$.

2. Determine whether each statement is always, sometimes, or never true.
   a. If a triangle has three congruent sides, then it has three congruent angles.
   b. If a triangle is isosceles, then it is equilateral.
   c. If a right triangle is isosceles, then it is equilateral.
   d. The largest angle of an isosceles triangle is obtuse.
   e. If a right triangle has a 45° angle, then it is isosceles.
   f. If an isosceles triangle has three acute angles, then it is equilateral.
   g. The vertex angle of an isosceles triangle is the largest angle of the triangle.

3. Give the measures of the three angles of each triangle.
   a. an equilateral triangle
   b. an isosceles right triangle
   c. an isosceles triangle in which the measure of the vertex angle is 70
   d. an isosceles triangle in which the measure of a base angle is 70
   e. an isosceles triangle in which the measure of the vertex angle is twice the measure of one of the base angles

Helping You Remember

4. If a theorem and its converse are both true, you can often remember them most easily by combining them into an “if-and-only-if” statement. Write such a statement for the Isosceles Triangle Theorem and its converse.
4-6 Enrichment

Triangle Challenges

Some problems include diagrams. If you are not sure how to solve the problem, begin by using the given information. Find the measures of as many angles as you can, writing each measure on the diagram. This may give you more clues to the solution.

1. Given: $BE = BF$, $\angle BFG \cong \angle BEF \cong \angle BED$, $m \angle BFE = 82$ and $ABFG$ and $BCDE$ each have opposite sides parallel and congruent.
   Find $m \angle ABC$.

2. Given: $AC = AD$, and $\overline{AB} \perp \overline{BD}$, $m \angle DAC = 44$ and $CE$ bisects $\angle ACD$.
   Find $m \angle DEC$.

3. Given: $m \angle UZY = 90$, $m \angle ZWX = 45$, $\triangle YZU \cong \triangle VWX$, $UVXY$ is a square (all sides congruent, all angles right angles).
   Find $m \angle WZY$.

4. Given: $m \angle N = 120$, $\overline{JN} \cong \overline{MN}$, $\triangle JNM \cong \triangle KLM$.
   Find $m \angle JKM$. 
Position and Label Triangles  A coordinate proof uses points, distances, and slopes to prove geometric properties. The first step in writing a coordinate proof is to place a figure on the coordinate plane and label the vertices. Use the following guidelines.

1. Use the origin as a vertex or center of the figure.
2. Place at least one side of the polygon on an axis.
3. Keep the figure in the first quadrant if possible.
4. Use coordinates that make the computations as simple as possible.

**Example**  Position an equilateral triangle on the coordinate plane so that its sides are \( a \) units long and one side is on the positive \( x \)-axis.

Start with \( R(0, 0) \). If \( RT \) is \( a \), then another vertex is \( T(a, 0) \).

For vertex \( S \), the \( x \)-coordinate is \( \frac{a}{2} \). Use \( b \) for the \( y \)-coordinate, so the vertex is \( S\left( \frac{a}{2}, b \right) \).

**Exercises**

Find the missing coordinates of each triangle.

1. 

   \[ \begin{array}{c}
   \text{A}(0, 0) \\
   \text{B}(2p, 0) \\
   \text{C} (?, q)
   \end{array} \]

2. 

   \[ \begin{array}{c}
   \text{R}(0, 0) \\
   \text{S}(2a, 0) \\
   \text{T} (?, ?)
   \end{array} \]

3. 

   \[ \begin{array}{c}
   \text{E}(?, ?) \\
   \text{G}(2g, 0) \\
   \text{F}(?, b)
   \end{array} \]

Position and label each triangle on the coordinate plane.

4. isosceles triangle \( \triangle RST \) with base \( RS \)
   
   4\( a \) units long

5. isosceles right \( \triangle DEF \)
   
   with legs \( e \) units long

6. equilateral triangle \( \triangle EQI \)
   
   with vertex \( Q(0, a) \) and sides 2\( b \) units long
Write Coordinate Proofs Coordinate proofs can be used to prove theorems and to verify properties. Many coordinate proofs use the Distance Formula, Slope Formula, or Midpoint Theorem.

Example Prove that a segment from the vertex angle of an isosceles triangle to the midpoint of the base is perpendicular to the base.

First, position and label an isosceles triangle on the coordinate plane. One way is to use \(T(a, 0), R(-a, 0),\) and \(S(0, c)\). Then \(U(0, 0)\) is the midpoint of \(RT\).

Given: Isosceles \(\triangle RST\); \(U\) is the midpoint of base \(RT\).

Prove: \(SU \perp RT\)

Proof:
\(U\) is the midpoint of \(RT\) so the coordinates of \(U\) are \(\left(\frac{-a + a}{2}, \frac{0 + 0}{2}\right) = (0, 0)\). Thus \(SU\) lies on the \(y\)-axis, and \(\triangle RST\) was placed so \(RT\) lies on the \(x\)-axis. The axes are perpendicular, so \(SU \perp RT\).

Exercises

Prove that the segments joining the midpoints of the sides of a right triangle form a right triangle.
Position and label each triangle on the coordinate plane.

1. right $\triangle FGH$ with legs $a$ units and $b$ units
2. isosceles $\triangle KLP$ with base $KP$ $6b$ units long
3. isosceles $\triangle AND$ with base $AD$ $5a$ long

Find the missing coordinates of each triangle.

4. $\triangle ABC$ with vertices $A(0,?), C(0,0), B(2a,0)$
5. $\triangle XYZ$ with vertices $Z(?,?), X(0,0), Y(2b,0)$
6. $\triangle MNO$ with vertices $M(?,?), O(0,0), N(3b,0)$
7. $\triangle PQR$ with vertices $R(2a,b), P(0,0), Q(?,?)$
8. $\triangle RST$ with vertices $R(?,?), N(0,0), P(7b,0)$
9. $\triangle TUV$ with vertices $T(?,?), S(-a,0), U(a,0)$

10. Write a coordinate proof to prove that in an isosceles right triangle, the segment from the vertex of the right angle to the midpoint of the hypotenuse is perpendicular to the hypotenuse.

   **Given:** isosceles right $\triangle ABC$ with $\angle ABC$ the right angle and $M$ the midpoint of $\overline{AC}$

   **Prove:** $BM \perp AC$
Position and label each triangle on the coordinate plane.

1. equilateral \( \triangle SWY \) with sides \( \frac{1}{4}a \) long

2. isosceles \( \triangle BLP \) with base \( BL \) 3b units long

3. isosceles right \( \triangle DGJ \) with hypotenuse \( DJ \) and legs 2a units long

Find the missing coordinates of each triangle.

4. \( S(?, ?) \)

5. \( E(0, ?) \)

6. \( M(0, ?) \)

**NEIGHBORHOODS** For Exercises 7 and 8, use the following information.
Karina lives 6 miles east and 4 miles north of her high school. After school she works part time at the mall in a music store. The mall is 2 miles west and 3 miles north of the school.

7. Write a coordinate proof to prove that Karina’s high school, her home, and the mall are at the vertices of a right triangle.

   **Given:** \( \triangle SKM \)
   
   **Prove:** \( \triangle SKM \) is a right triangle.

8. Find the distance between the mall and Karina’s home.
Pre-Activity  How can the coordinate plane be useful in proofs?

Read the introduction to Lesson 4-7 at the top of page 222 in your textbook.

From the coordinates of $A$, $B$, and $C$ in the drawing in your textbook, what do you know about $\triangle ABC$?

Reading the Lesson

1. Find the missing coordinates of each triangle.
   a. $R(?, b)$
   b. $E(?, a)$

2. Refer to the figure.
   a. Find the slope of $SR$ and the slope of $ST$.
   b. Find the product of the slopes of $SR$ and $ST$. What does this tell you about $SR$ and $ST$?
   c. What does your answer from part b tell you about $\triangle RST$?
   d. Find $SR$ and $ST$. What does this tell you about $SR$ and $ST$?
   e. What does your answer from part d tell you about $\triangle RST$?
   f. Combine your answers from parts c and e to describe $\triangle RST$ as completely as possible.
   g. Find $m\angle SRT$ and $m\angle STR$.
   h. Find $m\angle OSR$ and $m\angle OST$.

Helping You Remember

3. Many students find it easier to remember mathematical formulas if they can put them into words in a compact way. How can you use this approach to remember the slope and midpoint formulas easily?
How Many Triangles?

Each puzzle below contains many triangles. Count them carefully. Some triangles overlap other triangles.

How many triangles are there in each figure?

1. 2. 3.

4. 5. 6.

How many triangles can you form by joining points on each circle? List the vertices of each triangle.

7. 8.

9.
Chapter 4 Test, Form 1

Write the letter for the correct answer in the blank at the right of each question.

1. How would this triangle be classified by angles?
   A. acute          B. equiangular
   C. obtuse         D. right

2. What is the value of \( x \) if \( \triangle ABC \) is equilateral?
   A. \( \frac{1}{8} \)          B. \( \frac{1}{2} \)
   C. \( 8 \)                 D. \( 1 \)

Use the figure for Questions 3–4 and write the letter for the correct answer in the blank at the right of each question.

3. What is \( m\angle 2? \)
   A. 50          B. 70          C. 110          D. 120

4. What is \( m\angle 4? \)
   A. 10          B. 60          C. 100          D. 120

5. What are the congruent triangles in the diagram?
   A. \( \triangle ABC \cong \triangle EBD \)
   B. \( \triangle ABE \cong \triangle CBD \)
   C. \( \triangle AEB \cong \triangle CBD \)
   D. \( \triangle ABE \cong \triangle CDB \)

6. If \( \triangle CJW \cong \triangle AGS \), \( m\angle A = 50 \), \( m\angle J = 45 \),
   and \( m\angle S = 16x + 5 \), what is \( x? \)
   A. 17.5          B. 11.875
   C. 6            D. 5

7. Which postulate can be used to prove the triangles congruent?
   A. SSS          B. SAS          C. ASA          D. AAS

8. What reason should be given for statement 5 in the proof?
   \( \text{Given: } \overline{DB} \text{ is the perpendicular bisector of } \overline{AC}. \)
   \( \text{Prove: } \triangle ADB \cong \triangle CDB \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
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</thead>
<tbody>
<tr>
<td>1. ( \overline{DB} ) is the perpendicular bisector of ( \overline{AC} ).</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \overline{AB} \cong \overline{CB} )</td>
<td>2. Midpoint Theorem</td>
</tr>
<tr>
<td>3. ( \angle ABD \cong \angle CBD )</td>
<td>3. ( \perp \text{ line; all right } \angle \text{s are } \cong. )</td>
</tr>
<tr>
<td>4. ( \overline{DB} \cong \overline{DB} )</td>
<td>4. Reflexive Property</td>
</tr>
<tr>
<td>5. ( \triangle ADB \cong \triangle CDB )</td>
<td>5. ?</td>
</tr>
</tbody>
</table>

A. SSS          B. AAS          C. ASA          D. SAS
Use the proof for Questions 9–10 and write the letter for the correct answer in the blank at the right of each question.

**Given:** \( L \) is the midpoint of \( JM; \ JK \parallel NM \).

**Prove:** \( \triangle JKL \cong \triangle MNL \)

**Statements** | **Reasons**
--- | ---
1. \( L \) is the midpoint of \( JM \). | 1. Given
2. \( JL \equiv ML \) | 2. Definition of midpoint
3. \( JK \parallel MN \) | 3. Given
4. \( \angle JKL \equiv \angle MNL \) | 4. Alt. int. \( \triangle \) are \( \equiv \).
5. \( \angle JLK \equiv \angle MLN \) | 5. (Question 9)
6. \( \triangle JKL \equiv \triangle MNL \) | 6. (Question 10)

9. What is the reason for \( \angle JLK \equiv \angle MLN \)?
   - A. definition of midpoint
   - B. corresponding angles
   - C. vertical angles
   - D. alternate interior angles

10. What is the reason for \( \triangle JKL \equiv \triangle MNL \)?
    - A. AAS
    - B. ASA
    - C. SAS
    - D. SSS

Use the figure for Questions 11–12 and write the letter for the correct answer in the blank at the right of each question.

11. If \( \triangle LNM \) is isosceles and \( T \) is the midpoint of \( LN \), which postulate can be used to prove \( \triangle MLT \equiv \triangle MNT \)?
    - A. AAA
    - B. AAS
    - C. SAS
    - D. ABC

12. If \( \triangle MLT \equiv \triangle MNT \), what is used to prove \( \angle 1 \equiv \angle 2 \)?
    - A. CPCTC
    - B. definition of isosceles triangle
    - C. definition of perpendicular
    - D. definition of angle bisector

13. What are the missing coordinates of this triangle?
    - A. \((2a, 2c)\)
    - B. \((2a, 0)\)
    - C. \((0, 2a)\)
    - D. \((a, 2c)\)

**Bonus** What is the classification by sides of a triangle with coordinates \( A(5, 0), B(0, 5), \) and \( C(-5, 0) \)?

B: _____________
Chapter 4 Test, Form 2A

Write the letter for the correct answer in the blank at the right of each question.

1. What is the length of the sides of this equilateral triangle?
   A. 42             B. 30
   C. 15             D. 12

2. What is the classification of $\triangle ABC$ with vertices $A(4, 1), B(2, -1)$, and $C(-2, -1)$ by its sides?
   A. equilateral    B. isosceles
   C. scalene        D. right

Use the figure for Questions 3–4 and write the letter for the correct answer in the blank at the right of each question.

3. What is $m\angle 1$?
   A. 40             B. 50
   C. 70             D. 90

4. What is $m\angle 3$?
   A. 40             B. 70
   C. 90             D. 110

5. If $\triangle DKL \cong \triangle EGS$, which segment in $\triangle EGS$ corresponds to $\overline{DL}$?
   A. $\overline{EG}$   B. $\overline{ES}$
   C. $\overline{GS}$   D. $\overline{GE}$

6. Which triangles are congruent in the figure?
   A. $\triangle KLM \cong \triangle JMN$     B. $\triangle JKL \cong \triangle JML$
   C. $\triangle JKL \cong \triangle LNM$     D. $\triangle JKL \cong \triangle MNL$

Use the proof for Questions 7–8 and write the letter for the correct answer in the blank at the right of each question.

Given: $\overline{RJ} \parallel \overline{EI}; RI$ bisects $\overline{JE}$.
Prove: $\triangle R\!J\!N \cong \triangle I\!E\!N$

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\overline{RJ} \parallel \overline{IE}$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $\angle R!J!N \cong \angle I!E!N$</td>
<td>2. (Question 7)</td>
</tr>
<tr>
<td>3. $\overline{RI}$ bisects $\overline{JE}$.</td>
<td>3. Given</td>
</tr>
<tr>
<td>4. $\overline{J!N} \cong \overline{E!N}$</td>
<td>4. Definition of bisector</td>
</tr>
<tr>
<td>5. $\angle R!J!N \cong \angle I!E!N$</td>
<td>5. Vert. $\triangle$ are $\cong$</td>
</tr>
<tr>
<td>6. $\triangle R!J!N \cong \triangle I!E!N$</td>
<td>6. (Question 8)</td>
</tr>
</tbody>
</table>

7. What is the reason for statement 2 in the proof?
   A. Isosceles Triangle Theorem     B. same side interior angles
   C. corresponding angles           D. Alternate Interior Angle Theorem

8. What is the reason for statement 6?
   A. ASA     B. AAS
   C. SAS     D. SSS
9. If \( \triangle ABC \) is isosceles and \( \overline{AE} \cong \overline{FC} \), which theorem or postulate can be used to prove \( \triangle AEB \cong \triangle CFB \)?
   A. SSS  
   B. SAS  
   C. ASA  
   D. AAS

Use the proof for Questions 10–11 and write the letter for the correct answer in the blank at the right of each question.

Given: \( \overline{DA} \parallel \overline{YN} \); \( \overline{DA} \cong \overline{YN} \)
Prove: \( \triangle NDY \cong \triangle DNA \)

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>1. ( \overline{DA} \parallel \overline{YN} )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \triangle ADN \cong \triangle YND )</td>
<td>2. Alt. int. ( \angle ) are ( \cong ).</td>
</tr>
<tr>
<td>3. ( \overline{DA} \cong \overline{YN} )</td>
<td>3. Given</td>
</tr>
<tr>
<td>4. ( \overline{DN} \cong \overline{DN} )</td>
<td>4. Reflexive Property</td>
</tr>
<tr>
<td>5. ( \triangle NDY \cong \triangle DNA )</td>
<td>5. (Question 10)</td>
</tr>
<tr>
<td>6. ( \angle NDY \cong \angle DNA )</td>
<td>6. (Question 11)</td>
</tr>
</tbody>
</table>

10. What is the reason for statement 5?
   A. ASA  
   B. AAS  
   C. SAS  
   D. SSS

11. What is the reason for statement 6?
   A. Alt. int. \( \angle \)s are \( \cong \).  
   B. CPCTC  
   C. Corr. angles are \( \cong \).  
   D. Isosceles Triangle Theorem

12. What is the classification of a triangle with vertices \( A(3, 3), B(6, -2), C(0, -2) \) by its sides?
   A. isosceles  
   B. scalene  
   C. equilateral  
   D. right

13. What are the missing coordinates of the triangle?
   A. \((-2b, 0)\)  
   B. \((0, 2b)\)  
   C. \((-c, 0)\)  
   D. \((0, -c)\)

Bonus Name the coordinates of points \( A \) and \( C \) in isosceles right \( \triangle ABC \) if point \( C \) is in the second quadrant.
Chapter 4 Test, Form 2B

Write the letter for the correct answer in the blank at the right of each question.

1. What is the length of the sides of this equilateral triangle?
   A. 2.5  
   B. 5  
   C. 15  
   D. 20

2. What is the classification of $\triangle ABC$ with vertices $A(0, 0), B(4, 3),$ and $C(4, -3)$ by its sides?
   A. equilateral  
   B. isosceles  
   C. scalene  
   D. right

Use the figure for Questions 3–4 and write the letter for the correct answer in the blank at the right of each question.

3. What is $m\angle 1$?
   A. 120  
   B. 90  
   C. 60  
   D. 30

4. What is $m\angle 2$?
   A. 120  
   B. 90  
   C. 60  
   D. 30

5. If $\triangle TGS \cong \triangle KEL$, which angle in $\triangle KEL$ corresponds to $\angle T$?
   A. $\angle L$  
   B. $\angle E$  
   C. $\angle K$  
   D. $\angle A$

6. Which triangles are congruent in the figure?
   A. $\triangle HMN \cong \triangle HGN$  
   B. $\triangle HMN \cong \triangle NGH$  
   C. $\triangle NMH \cong \triangle NGH$  
   D. $\triangle MHN \cong \triangle HGN$

Use the proof for Questions 7–8 and write the letter for the correct answer in the blank at the right of each question.

Given: $AB \parallel CD; AC$ bisects $BD$.
Prove: $\triangle ABE \cong \triangle CDE$

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>1. $AC$ bisects $BD$.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $BE \cong DE$</td>
<td>2. (Question 7)</td>
</tr>
<tr>
<td>3. $AB \parallel CD$</td>
<td>3. Given</td>
</tr>
<tr>
<td>4. $\triangle ABE \cong \triangle CDE$</td>
<td>4. Alt. int. $\angle$ are $\cong$</td>
</tr>
<tr>
<td>5. (Question 8)</td>
<td>5. Vert. $\angle$ are $\cong$</td>
</tr>
<tr>
<td>6. $\triangle ABE \cong \triangle CDE$</td>
<td>5. ASA</td>
</tr>
</tbody>
</table>

7. What is the reason for statement 2?  
   A. Definition of bisector  
   B. Midpoint Theorem  
   C. Given  
   D. Alternate Interior Angle Theorem

8. What is the statement for reason 5?  
   A. $\angle BEA \cong \angle DEC$  
   B. $\angle ABE \cong \angle CDE$  
   C. $\angle EAB \cong \angle ECD$  
   D. $\angle BEC \cong \angle DEA$
9. If $\overline{AF} \cong \overline{DE}$, $\overline{AB} \cong \overline{FC}$ and $\overline{AB} \parallel \overline{FC}$, which theorem or postulate can be used to prove $\triangle ABE \cong \triangle FCD$?
   - A. AAS
   - B. ASA
   - C. SAS
   - D. SSS

Use the proof for Questions 10–11 and write the letter for the correct answer in the blank at the right of each question.

Given: $\overline{EG} \cong \overline{IA}$; $\angle EGA \cong \angle IAG$
Prove: $\angle GEN \cong \angle AIN$

<table>
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<td>1. $\overline{EG} \cong \overline{IA}$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $\angle EGA \cong \angle IAG$</td>
<td>2. Given</td>
</tr>
<tr>
<td>3. $\overline{GA} \cong \overline{GA}$</td>
<td>3. Reflexive Property</td>
</tr>
<tr>
<td>4. $\triangle EGA \cong \triangle IAG$</td>
<td>4. (Question 10)</td>
</tr>
<tr>
<td>5. $\angle GEN \cong \angle AIN$</td>
<td>5. (Question 11)</td>
</tr>
</tbody>
</table>

10. What is the reason for statement 4?
   - A. SSS
   - B. ASA
   - C. SAS
   - D. AAS

11. What is the reason for statement 5?
   - A. Alt. int. $\angle$ are $\cong$.
   - B. Same Side Interior Angles
   - C. Corr. angles are $\cong$.
   - D. CPCTC

12. What is the classification of a triangle with vertices $A(-3, -1)$, $B(-2, 2)$, $C(3, 1)$ by its sides?
   - A. scalene
   - B. isosceles
   - C. equilateral
   - D. right

13. What are the missing coordinates of the triangle?
   - A. $(a, 0)$
   - B. $(b, 0)$
   - C. $(c, 0)$
   - D. $(0, c)$

Bonuses

Find $x$ in the triangle.
Chapter 4 Test, Form 2C

1. Use a protractor and ruler to classify the triangle by its angles and sides.

2. Find \( x \), \( AB \), \( BC \), and \( AC \) if \( \triangle ABC \) is equilateral.

3. Find the measure of the sides of the triangle if the vertices of \( \triangle EFG \) are \( E(-3, 3) \), \( F(1, -1) \), and \( G(-3, -5) \). Then classify the triangle by its sides.

Find the measure of each angle.

4. \( m\angle 1 \)

5. \( m\angle 2 \)

6. \( m\angle 3 \)

7. Identify the congruent triangles and name their corresponding congruent angles.

8. Verify that \( \triangle ABC \cong \triangle A'B'C' \) preserves congruence, assuming that corresponding angles are congruent.
9. \(ABCD\) is a quadrilateral with \(AB \cong CD\) and \(AB \parallel CD\). Name the postulate that could be used to prove \(\triangle BAC \cong \triangle DCA\). Choose from SSS, SAS, ASA, and AAS.

10. \(\triangle KLM\) is an isosceles triangle and \(\angle 1 \cong \angle 2\). Name the theorem that could be used to determine \(\angle LKP \cong \angle LMN\). Then name the postulate that could be used to prove \(\triangle LKP \cong \triangle LMN\). Choose from SSS, SAS, ASA, and AAS.

11. Use the figure to find \(m\angle 1\).

12. Find \(x\).

13. Position and label isosceles \(\triangle ABC\) with base \(\overline{AB}\) \(b\) units long on the coordinate plane.

14. \(\overline{CP}\) joins point \(C\) in isosceles right \(\triangle ABC\) to the midpoint \(P\), of \(\overline{AB}\). Name the coordinates of \(P\). Then determine the relationship between \(\overline{AB}\) and \(\overline{CP}\).

**Bonus** Without finding any other angles or sides congruent, which pair of triangles can be proved to be congruent by the HL Theorem?
1. Use a protractor and ruler to classify the triangle by its angles and sides.

2. Find $x$, $AB$, $BC$, $AC$ if $\triangle ABC$ is isosceles.

3. Find the measure of the sides of the triangle if the vertices of $\triangle EFG$ are $E(1, 4)$, $F(5, 1)$, and $G(2, -3)$. Then classify the triangle by its sides.

Find the measure of each angle.

4. $m\angle 1$

5. $m\angle 2$

6. $m\angle 3$

7. Identify the congruent triangles and name their corresponding congruent angles.

8. Verify that $\triangle JKL \cong \triangle J'K'L'$ preserves congruence, assuming that corresponding angles are congruent.

9. In quadrilateral $JKLM$, $JK \equiv LK$ and $MK$ bisects $\angle LKJ$. Name the postulate that could be used to prove $\triangle MKL \cong \triangle MKJ$. Choose from SSS, SAS, ASA, and AAS.
10. \( \triangle ABC \) is an isosceles triangle with \( BD \perp AC \). Name the theorem that could be used to determine \( \angle A \equiv \angle C \). Then name the postulate that could be used to prove \( \triangle BDA \equiv \triangle BDC \). Choose from SSS, SAS, ASA, and AAS.

11. Use the figure to find \( m\angle 1 \).

12. Find \( x \).

13. Position and label equilateral \( \triangle KLM \) with side lengths 3\( a \) units long on the coordinate plane.

14. \( MN \) joins the midpoint of \( AB \) and the midpoint of \( AC \) in \( \triangle ABC \). Find the coordinates of \( M \) and \( N \), and the slopes of \( MN \) and \( BC \).

**Bonus** Without finding any other angles or sides congruent, which pair of triangles can be proved to be congruent by the LL Theorem?

---

**B:** 

---

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1. If $\triangle ABC$ is isosceles, $\angle B$ is the vertex angle, $AB = 20x - 2$, $BC = 12x + 30$, and $AC = 25x$, find $x$ and the measure of each side of the triangle.

2. Given $A(0, 4), B(5, 4),$ and $C(-3, -2)$, find the measure of the sides of the triangle. Then classify the triangle by its sides and angles.

Use the figure to answer Questions 3–5.

3. Find $x$.

4. $m\angle 1$, if $m\angle 1 = 4x + 10$.

5. $m\angle 2$

6. Verify that the following preserves congruence, assuming that corresponding angles are congruent. $\triangle ABC$ is reflected over the $x$-axis as follows.

$A(-1, 1) \rightarrow A'(1, -1)$
$B(4, 2) \rightarrow B'(4, -2)$
$C(1, 5) \rightarrow C'(1, -5)$

Verify $\triangle ABC \cong \triangle A'B'C'$.

7. Determine whether $\triangle GHI \cong \triangle JKL$, given $G(1, 2), H(5, 4), I(3, 6)$ and $J(-4, -5), K(0, -3), L(-2, -1)$. Explain.

8. In the figure, $\overline{AC} \cong \overline{FD}, \overline{AB} \parallel \overline{DE}$, and $\overline{AC} \parallel \overline{FD}$. Name the postulate that could be used to prove $\triangle ABC \cong \triangle DEC$. Choose from SSS, SAS, ASA, and AAS.
For Questions 9 and 10, complete this two-column proof.

Given: \( \triangle ABC \) is an isosceles triangle with base \( AC \).
\( D \) is the midpoint of \( AC \).

Prove: \( BD \) bisects \( \angle ABC \).

<table>
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<tbody>
<tr>
<td>1. ( \triangle ABC ) is isosceles with base ( AC ).</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( AB \cong CB )</td>
<td>2. Def. of isosceles triangle.</td>
</tr>
<tr>
<td>3. ( \angle A \cong \angle C )</td>
<td>3. (Question 9)</td>
</tr>
<tr>
<td>4. ( D ) is the midpoint of ( AC ).</td>
<td>4. Given</td>
</tr>
<tr>
<td>5. ( AD \cong CD )</td>
<td>5. Midpoint Theorem</td>
</tr>
<tr>
<td>6. ( \triangle ABD \cong \triangle CBD )</td>
<td>6. (Question 10)</td>
</tr>
<tr>
<td>7. ( \angle 1 \cong \angle 2 )</td>
<td>7. CPCTC</td>
</tr>
<tr>
<td>8. ( BD ) bisects ( \angle ABC ).</td>
<td>8. Def. of angle bisector</td>
</tr>
</tbody>
</table>

9. \[ \text{__________} \]
10. \[ \text{__________} \]

11. Find \( x \).

12. Position and label isosceles \( \triangle ABC \) with base \( AB \) \((a + b)\) units long on a coordinate plane

**Bonus** In the figure, \( \triangle ABC \) is isosceles, \( \triangle ADC \) is equilateral, \( \triangle AEC \) is isosceles, and the measures of \( \angle 9, \angle 1, \) and \( \angle 3 \) are all equal. Find the measures of the nine numbered angles.

**B:** \[ \text{__________} \]
Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solution in more than one way or investigate beyond the requirements of the problem.

1. \( (20x - 10)^\circ \) 
   \( (9x + 4)^\circ \)
   a. Classify the triangle by its angles and sides.
   b. Show the steps needed to solve for \( x \).

2. a. Describe how to determine whether a triangle with coordinates \( A(1, 4) \), \( B(1, -1) \), and \( C(-4, 4) \) is an equilateral triangle.
   b. Is the triangle equilateral? Explain.

3. Explain how to find \( m\angle 1 \) and \( m\angle 2 \) in the figure.

4. a. State the theorem or postulate that can be used to prove that the triangles are congruent.
   b. List their corresponding congruent angles and sides.

5. Given: \( AB \parallel DE, AD \) bisects \( BE \).
   Prove: \( \triangle ABC \cong \triangle DEC \) by using the ASA postulate.
Choose from the terms above to complete each sentence.

1. A triangle that is equilateral is also called a(n) ___.

2. A(n) ___ has at least one obtuse angle.

3. The sum of the ___ is equivalent to the exterior angle of a triangle.

4. The ___ angles of an isosceles triangle are congruent.

5. A triangle with different measures for each side is classified as a(n) ___.

6. A ___ organizes a series of statements in logical order written in boxes and uses arrows to indicate the order of the statements.

7. A triangle that is translated, reflected or rotated and preserves its shape, is said to be a(n) ___.

8. The ASA postulate involves two corresponding angles and their corresponding ___.

9. A ___ uses figures in the coordinate plane and algebra to prove geometric concepts.

10. The ___ is formed by the congruent legs of an isosceles triangle.

In your own words—

11. corollary

12. congruent triangles

13. acute triangle
Chapter 4 Quiz
(Lessons 4–1 and 4–2)

1. Use a protractor to classify the triangle by its angles and sides.

2. STANDARDIZED TEST PRACTICE What is the best classification of this triangle by its angles and sides?
   A. acute isosceles
   B. right isosceles
   C. obtuse isosceles
   D. obtuse equilateral

3. If \( \triangle ABC \) is an isosceles triangle, \( \angle B \) is the vertex angle, \( AB = 6x + 3, BC = 8x - 1 \), and \( AC = 10x - 10 \), find \( x \) and the measures of each side of the triangle.

4. If \( A(1, 5), B(3, -2), \) and \( C(-3, 0) \), find the measures of the sides of \( \triangle ABC \). Then classify the triangle by its sides.

Find the measure of each angle in the figure.
5. \( m\angle 1 \)
6. \( m\angle 2 \)
7. \( m\angle 3 \)
8. \( m\angle 4 \)
9. \( m\angle 5 \)
10. \( m\angle 6 \)

Chapter 4 Quiz
(Lessons 4–3 and 4–4)

1. Identify the congruent triangles in the figure.

2. STANDARDIZED TEST PRACTICE If \( \triangle JGO \cong \triangle RWI \), which angle corresponds to \( \angle I? \)
   A. \( \angle J \)
   B. \( \angle R \)
   C. \( \angle G \)
   D. \( \angle O \)

3. Verify that the following preserves congruence assuming that corresponding angles are congruent. \( \triangle ABC \cong \triangle A'B'C' \)

4. In quadrilateral \( EFGH, \overline{FG} \cong \overline{HE}, \) and \( FG \parallel HE \). Name the postulate that could be used to prove \( \triangle EHF \cong \triangle GFH \). Choose from SSS, SAS, ASA, and AAS.
Chapter 4 Quiz
(Lessons 4–5 and 4–6)

For Questions 1 and 2, complete the two-column proof by supplying the missing information for each corresponding location.

Given: \( \angle Z \equiv \angle C; \overline{AK} \) bisects \( \angle ZKC \).

Prove: \( \triangle AKZ \equiv \triangle AKC \)

<table>
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<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \angle Z \equiv \angle C; \overline{AK} ) bisects ( \angle ZKC ).</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \angle ZKA \equiv \angle CKA )</td>
<td>2. (Question 1)</td>
</tr>
<tr>
<td>3. ( \overline{AK} \equiv \overline{AK} )</td>
<td>3. Reflexive Property</td>
</tr>
<tr>
<td>4. ( \triangle AKZ \equiv \triangle AKC )</td>
<td>4. (Question 2)</td>
</tr>
</tbody>
</table>

Refer to the figure for Questions 3 and 4.

3. Find \( m \angle 1 \).

4. Find \( m \angle 2 \).

Chapter 4 Quiz
(Lesson 4–7)

1. Find the missing coordinates.

Position and label each triangle on a coordinate plane.

2. Right \( \triangle DJL \) with hypotenuse \( \overline{DJ} \); \( LJ = \frac{1}{2} DL \) and \( \overline{DL} \) is \( a \) units long.

3. Isosceles \( \triangle EGS \) with base \( \overline{ES} \); \( \frac{1}{2} b \) units long

For Questions 4 and 5, complete the coordinate proof by supplying the missing information for each corresponding location.

Given: \( \triangle ABC \) with \( A(-1, 1), B(5, 1), \) and \( C(2, 6) \).

Prove: \( \triangle ABC \) is isosceles.

By the Distance Formula the lengths of the three sides are as follows: (Question 4). Since (Question 5), \( \triangle ABC \) is isosceles.
Chapter 4 Mid-Chapter Test
(Lessons 4–1 through 4–3)

Part I  Write the letter for the correct answer in the blank at the right of each question.

1. What is the best classification for this triangle?
   A. acute scalene  
   B. obtuse equilateral  
   C. acute isosceles  
   D. obtuse isosceles  

Find the missing angle measures.

2. What is \( m\angle 1 \)?
   A. 50  
   B. 60  
   C. 100  
   D. 105  

3. What is \( m\angle 2 \)?
   A. 40  
   B. 50  
   C. 60  
   D. 100  

4. If \( \triangle SJL \cong \triangle DMT \), which segment in \( \triangle DMT \) corresponds to \( \overline{LS} \) in \( \triangle SJL \)?
   A. \( \overline{DT} \)  
   B. \( \overline{TD} \)  
   C. \( \overline{MD} \)  
   D. \( \overline{MT} \)  

Part II

5. Find the measures of the sides of \( \triangle ABC \) and classify it by its sides. \( A(1, 3), B(5, -2), \) and \( C(0, -4) \)  

6. In \( \triangle ABC \) and \( \triangle A'B'C' \), \( \angle A \cong \angle A' \), \( \angle B \cong \angle B' \), and \( \angle C \cong \angle C' \). Find the lengths needed to prove \( \triangle ABC \cong \triangle A'B'C' \).  

7. What information would you need to know about \( \overline{PO} \) and \( \overline{LN} \) for \( \triangle LMP \) to be congruent to \( \triangle NMO \) by SSS?
1. Name the geometric figure that is modeled by the second hand of a clock. (Lesson 1-1)

2. Find the precision for a measurement of 36 inches. (Lesson 1-2)

For Questions 3–5, use the number line.

\[
\begin{array}{cccccc}
& A & B & C & D & E \\
-10 & -9 & -8 & -7 & -6 & -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 & 6
\end{array}
\]

3. Find \(BC\). (Lesson 1-3)

4. Find the coordinate of the midpoint of \(AD\). (Lesson 1-3)

5. If \(B\) is the midpoint of a segment having one endpoint at \(E\), what is the coordinate of its other endpoint? (Lesson 1-3)

For Questions 6 and 7, determine whether each statement is always, sometimes, or never true. Explain your answer.

(Lesson 2-5)

6. If \(DE \cong EF\), then \(E\) is the midpoint of \(DF\).

7. If points \(A\) and \(B\) lie in plane \(Q\), then \(AB\) lies in \(Q\).

8. Find the slope of a line parallel to \(x = 2\). (Lesson 3-3)

9. Find the distance between \(y = -9\) and \(y = -5\). (Lesson 3-6)

For Questions 10–12, use the figure.

10. Name the segment that represents the distance from \(F\) to \(AD\). (Lesson 3-6)

11. Classify \(\triangle ADC\). (Lesson 4-1)

12. Find \(m\angle ACD\). (Lesson 4-2)

13. Name the corresponding congruent angles and sides for \(\triangle PQR \cong \triangle HGB\). (Lesson 4-3)

14. If \(\angle QRP \cong \angle SRT\), and \(R\) is the midpoint of \(PT\), which theorem or postulate can be used to prove \(\triangle QRP \cong \triangle SRT\)? Choose from SSS, SAS, ASA, and AAS. (Lesson 4-5)

15. Name the missing coordinates of \(\triangle GEF\). (Lesson 4-7)
Part 1: Multiple Choice

Instructions: Fill in the appropriate oval for the best answer.

1. If $m\angle 1 = 5x - 4$, and $m\angle 2 = 52 - 9y$, which values for $x$ and $y$ would make $\angle 1$ and $\angle 2$ complementary? (Lesson 1-5)
   A. $x = 2, y = 12$
   B. $x = 12, y = 2$
   C. $x = 27, y = \frac{1}{3}$
   D. $x = \frac{1}{3}, y = 27$

2. Which is not a polygon? (Lesson 1-6)
   E. F. G. H.

3. Complete the statement so that its conditional and its converse are true.
   If $\angle 1 \equiv \angle 2$, then $\angle 1$ and $\angle 2$ _____. (Lesson 2-3)
   A. are supplementary.
   B. are complementary.
   C. have the same measure.
   D. are alternate interior angles.

4. Complete this proof. (Lesson 2-7)
   Given: $UV \equiv VW$
   $VW \equiv WX$
   Prove: $UV = WX$
   Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
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<tbody>
<tr>
<td>1. $UV \equiv VW; VW \equiv WX$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $UV = VW; VW = WX$</td>
<td>2. <em><strong><strong>?</strong></strong></em>_</td>
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<tr>
<td>3. $UV = WX$</td>
<td>3. Transitive Property</td>
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</table>

   E. Definition of congruent segments
   F. Substitution Property
   G. Segment Addition Postulate
   H. Symmetric Property

5. Which equation has a slope of $\frac{1}{3}$ and a $y$-intercept of $-2$? (Lesson 3-4)
   A. $y = \frac{1}{3}x + 2$
   B. $y = \frac{1}{3}x - 2$
   C. $y = 2x - \frac{1}{3}$
   D. $y = -2x + \frac{1}{3}$

6. Classify $\triangle DEF$ with vertices $D(2, 3), E(5, 7)$ and $F(9, 4)$. (Lesson 4-1)
   E. acute
   F. equiangular
   G. obtuse
   H. right

7. Which postulate or theorem can be used to prove $\triangle ABD \equiv \triangle CBD$? (Lesson 4-4)
   A. SAS
   B. SSS
   C. ASA
   D. AAS
8. What is the y-coordinate of the midpoint of
   A(12, 6) and B(−15, −6)? (Lesson 1-3)

9. If $m\angle 1 = 112$, find $m\angle 10$.
   (Lesson 3-2)

10. If $JK \parallel LM$, then $\angle 4$ must
    be supplementary to $\angle \_ \_ \_ \_ \_$. (Lesson 3-5)

11. Find $PR$ if $\triangle PQR$ is isosceles, $\angle Q$ is the vertex
    angle, $PQ = 4x − 8$, $QR = x + 7$, and
    $PR = 6x − 12$. (Lesson 4-1)

12. The perimeter of a regular pentagon is 14.5 feet. If each side
    length of the pentagon is doubled, what is the new perimeter?
    (Lesson 1-6)

13. Make a conjecture about the next number in the sequence 5, 7,
    11, 17, 25. (Lesson 2-1)

14. Find $m\angle PQR$. (Lesson 4-2)

15. If $PQ = QS$, $QS = SR$, and $m\angle R = 20$, find $m\angle PSQ$.
    (Lesson 4-6)
Standardized Test Practice

Student Record Sheet (Use with pages 232–233 of the Student Edition.)

Part 1 Multiple Choice

Select the best answer from the choices given and fill in the corresponding oval.

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<td>8</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
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Part 2 Short Response/Grid In

Solve the problem and write your answer in the blank.

For Questions 12 and 14, also enter your answer by writing each number or symbol in a box. Then fill in the corresponding oval for that number or symbol.

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Part 3 Open-Ended

Record your answers for Questions 15–16 on the back of this paper.
Study Guide and Intervention
Classifying Triangles

Classify Triangles by Angles
One way to classify a triangle is by the measures of its angles.

• If one of the angles of a triangle is an obtuse angle, then the triangle is an obtuse triangle.
• If one of the angles of a triangle is a right angle, then the triangle is a right triangle.
• If all three of the angles of a triangle are acute angles, then the triangle is an acute triangle.

Classify Triangles by Sides
You can classify a triangle by the measures of its sides.

• Equal numbers of hash marks indicate congruent sides.

• If one of the angles of a triangle is an obtuse angle, then the triangle is an obtuse triangle.
• If all three sides of a triangle are congruent, then the triangle is an equilateral triangle.
• If one of the angles of a triangle is a right angle, then the triangle is a right triangle.
• If at least two sides of a triangle are congruent, then the triangle is an isosceles triangle.
• If all three angles of a triangle are acute angles, then the triangle is an acute triangle.
• If no two sides of a triangle are congruent, then the triangle is a scalene triangle.
• If all three angles of an acute triangle are congruent, then the triangle is an equiangular triangle.

Example
Classify each triangle.

a. The triangle has one angle that is obtuse. It is an obtuse triangle.

b. All three angles are congruent, so all three angles have measure 60°. The triangle is an equiangular triangle.

c. The triangle has one right angle. It is a right triangle.

Exercises
Classify each triangle as equilateral, isosceles, or scalene.

1. scalene
2. equilateral
3. scalene
4. isosceles
5. isosceles
6. equilateral

7. Find the measure of each side of equilateral \( \triangle RST \) with \( RS = 2x + 2, ST = 3x, \) and \( TR = 5x - 4 \).

8. Find the measure of each side of isosceles \( \triangle ABC \) with \( AB = BC \) if \( AB = 4y, BC = 3y + 2, \) and \( AC = 3y \).

9. Find the measure of each side of \( \triangle ABC \) with vertices \( A(-1, 5), B(6, 1), \) and \( C(2, -6) \). Classify the triangle.

   \( AB = BC = \sqrt{65}, AC = \sqrt{130}; \triangle ABC \) is isosceles.
4-1 Practice
Classifying Triangles

Use a protractor to classify each triangle as acute, equiangular, obtuse, or right.

1. equiangular
2. obtuse
3. right

Identify the indicated type of triangles.

4. acute
5. obtuse
6. acute

Identify the indicated type of triangles if $AB = AD = BD = DC$, $BE = ED$, $AB \perp BC$, and $ED \perp DC$.

7. right
8. isosceles

ALGEBRA Find $x$ and the measure of each side of the triangle.

9. $x = 6, AB = 16, BC = 16, CA = 16$
10. $x = 4, DE = 11, DF = 11, EF = 13$

Find the measures of the sides of $\triangle RST$ and classify each triangle by its sides.

11. $R(0, 2), S(2, 5), T(4, 2)$
   $RS = \sqrt{13}, ST = \sqrt{13}, RT = 4$; isosceles
12. $R(1, 3), S(4, 7), T(5, 4)$
   $RS = 5, ST = \sqrt{10}, RT = \sqrt{17}$; scalene
4-1 Reading to Learn Mathematics
Classifying Triangles

Pre-Activity

Why are triangles important in construction?

Read the introduction to Lesson 4-1 at the top of page 178 in your textbook.

- Why are triangles used for braces in construction rather than other shapes?
  Sample answer: Triangles lie in a plane and are rigid shapes.
- Why do you think that isosceles triangles are used more often than scalene triangles in construction? Sample answer: Isosceles triangles are symmetrical.

**Reading the Lesson**

1. Supply the correct numbers to complete each sentence.
   a. In an obtuse triangle, there are ___ acute angle(s), ___ right angle(s), and ___ obtuse angle(s).
   b. In an acute triangle, there are ___ acute angle(s), ___ right angle(s), and ___ obtuse angle(s).
   c. In a right triangle, there are ___ acute angle(s), ___ right angle(s), and ___ obtuse angle(s).

2. Determine whether each statement is always, sometimes, or never true.
   a. A right triangle is scalene. **sometimes**
   b. An obtuse triangle is isosceles. **sometimes**
   c. An equilateral triangle is a right triangle. **never**
   d. An equilateral triangle is isosceles. **always**
   e. An acute triangle is isosceles. **sometimes**
   f. A scalene triangle is obtuse. **sometimes**

3. Describe each triangle by as many of the following words as apply: acute, obtuse, right, scalene, isosceles, or equilateral.
   a. ___ acute, ___ scalene
   b. ___ obtuse, ___ isosceles
   c. ___ right, ___ scalene

Helping You Remember

4. A good way to remember a new mathematical term is to relate it to a nonmathematical definition of the same word. How is the use of the word acute, when used to describe acute pain, related to the use of the word acute when used to describe an acute angle or an acute triangle? Sample answer: Both are related to the meaning of acute as sharp. An acute pain is a sharp pain, and an acute angle can be thought of as an angle with a sharp point. In an acute triangle all of the angles are acute.

4-1 Enrichment

**Reading Mathematics**

When you read geometry, you may need to draw a diagram to make the text easier to understand.

**Example** Consider three points, A, B, and C on a coordinate grid.

The y-coordinates of A and B are the same. The x-coordinate of B is greater than the x-coordinate of A. Both coordinates of C are greater than the corresponding coordinates of B. Is triangle ABC acute, right, or obtuse?

To answer this question, first draw a sample triangle that fits the description.

Side AB must be a horizontal segment because the y-coordinates are the same. Point C must be located to the right and up from point B.

From the diagram you can see that triangle ABC must be obtuse.

**Answers**

1. Consider three points, R, S, and T on a coordinate grid. The x-coordinates of R and S are the same. The y-coordinate of T is between the y-coordinates of R and S. The x-coordinate of T is less than the x-coordinate of R. Is angle R of triangle RST acute, right, or obtuse? **acute**

2. Consider three noncollinear points, J, K, and L on a coordinate grid. The y-coordinates of J and K are the same. The x-coordinates of K and L are the same. Is triangle JKL acute, right, or obtuse? **right**

3. Consider three noncollinear points, D, E, and F on a coordinate grid.
   a. The x-coordinates of D and E are opposites. The y-coordinates of D and E are the same. The x-coordinate of F is 0. What kind of triangle must △DEF be: scalene, isosceles, or equilateral? **isosceles**

4. Consider three points, G, H, and I on a coordinate grid. Points G and H are on the positive y-axis, and the y-coordinate of G is twice the y-coordinate of H. Point I is on the positive x-axis, and the x-coordinate of I is greater than the y-coordinate of G. Is triangle GHI scalene, isosceles, or equilateral? **scalene**
Lesson 4-2

Study Guide and Intervention

Angles of Triangles

**Angle Sum Theorem**

If the measures of two angles of a triangle are known, the measure of the third angle can always be found.

Angle Sum Theorem

The sum of the measures of the angles of a triangle is 180°.

In the figure at the right, \( m \angle A + m \angle B + m \angle C = 180° \).

---

**Example 1**

Find \( m \angle T \).

\[
m \angle R + m \angle S + m \angle T = 180°
\]

\[
25 + 35 + m \angle T = 180°
\]

\[
60 + m \angle T = 180°
\]

\[
m \angle T = 120°
\]

---

**Example 2**

Find the missing angle measures.

\[
m \angle 1 + m \angle A + m \angle B = 180°
\]

\[
m \angle 1 + 58° + 90° = 180°
\]

\[
m \angle 1 + 148° = 180°
\]

\[
m \angle 1 = 32°
\]

\[
m \angle 2 = 32°
\]

---

**Exercises**

Find the measure of each numbered angle.

1. \( m \angle 1 = 28° \)

2. \( m \angle 1 = 120° \)

3. \( m \angle 1 = 30°, m \angle 2 = 60° \)

4. \( m \angle 1 = 56°, m \angle 2 = 56°, m \angle 3 = 74° \)

5. \( m \angle 1 = 30°, m \angle 2 = 60° \)

6. \( m \angle 1 = 8° \)

---

**Exterior Angle Theorem**

At each vertex of a triangle, the angle formed by one side and an extension of the other side is called an **exterior angle** of the triangle. For each exterior angle of a triangle, the **remote interior angles** are the interior angles that are not adjacent to that exterior angle. In the diagram below, \( \angle B \) and \( \angle A \) are the remote interior angles for exterior \( \angle DCB \).

**Exterior Angle Theorem**

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles.

\[
m \angle 1 = m \angle A + m \angle B
\]

---

**Example 1**

Find \( m \angle 1 \).

\[
m \angle 1 = m \angle R + m \angle S
\]

\[
60 + 80
\]

\[
= 140
\]

**Example 2**

Find \( x \).

\[
m \angle PQS = m \angle R + m \angle S
\]

\[
78 = 55 + x
\]

\[
23 = x
\]

---

**Exercises**

Find the measure of each numbered angle.

1. \( m \angle 1 = 115° \)

2. \( m \angle 1 = 60°, m \angle 2 = 120° \)

3. \( m \angle 1 = 60°, m \angle 2 = 60°, m \angle 3 = 120° \)

4. \( m \angle 1 = 109°, m \angle 2 = 29°, m \angle 3 = 71° \)

Find \( x \).

5. \( x = 25° \)

6. \( x = 29° \)
Find the missing angle measures.

1. $27$

Find the measure of each angle.

3. $m\angle 1 = 55$

4. $m\angle 2 = 55$

5. $m\angle 3 = 70$

Find the measure of each angle.

6. $m\angle 1 = 125$

7. $m\angle 2 = 55$

8. $m\angle 3 = 95$

Find the measure of each angle.

9. $m\angle 1 = 140$

10. $m\angle 2 = 40$

11. $m\angle 3 = 65$

12. $m\angle 4 = 75$

13. $m\angle 5 = 115$

Find the measure of each angle.

14. $m\angle 1 = 27$

15. $m\angle 2 = 27$

Find the missing angle measures.

1. $18$

2. $85$

Find the measure of each angle.

3. $m\angle 1 = 97$

4. $m\angle 2 = 83$

5. $m\angle 3 = 62$

Find the measure of each angle.

6. $m\angle 1 = 104$

7. $m\angle 2 = 45$

8. $m\angle 3 = 65$

9. $m\angle 2 = 79$

10. $m\angle 5 = 73$

11. $m\angle 6 = 147$

Find the measure of each angle if $\angle BAD$ and $\angle BDC$ are right angles and $m\angle ABC = 84$.

12. $m\angle 1 = 26$

13. $m\angle 2 = 32$

14. CONSTRUCTION The diagram shows an example of the Pratt Truss used in bridge construction. Use the diagram to find $m\angle 1$.

55
**Reading to Learn Mathematics**

**Angles of Triangles**

**Pre-Activity** How are the angles of triangles used to make kites?

Read the introduction to Lesson 4-2 at the top of page 185 in your textbook.

The frame of the simplest kind of kite divides the kite into four triangles. Describe these four triangles and how they are related to each other.

Sample answer: There are two pairs of right triangles that have the same size and shape.

**Reading the Lesson**

1. Refer to the figure.
   - a. Name the three interior angles of the triangle. (Use three letters to name each angle.) $\angle BAC$, $\angle ABC$, $\angle BCA$
   - b. Name three exterior angles of the triangle. (Use three letters to name each angle.) $\angle EAB$, $\angle DBC$, $\angle FCA$
   - c. Name the remote interior angles of $\angle EAB$, $\angle ABC$, $\angle BCA$
   - d. Find the measure of each angle without using a protractor.
      - i. $\angle BAC = 62$  ii. $\angle ABC = 118$  iii. $\angle ACF = 157$  iv. $\angle EAB = 141$

2. Indicate whether each statement is true or false. If the statement is false, replace the underlined word or number with a word or number that will make the statement true.
   - a. The acute angles of a right triangle are supplementary. false; complementary
   - b. The sum of the measures of the angles of any triangle is 180. false; 100
   - c. A triangle can have at most one right angle or acute angle. false; obtuse
   - d. If two angles of one triangle are congruent to two angles of another triangle, then the third angles of the triangles are congruent. true
   - e. The measure of an exterior angle of a triangle is equal to the difference of the measures of the two remote interior angles. false; sum
   - f. If the measures of two angles of a triangle are 62 and 93, then the measure of the third angle is 35. false; 25
   - g. An exterior angle of a triangle forms a linear pair with an interior angle of the triangle. true

**Helping You Remember**

3. Many students remember mathematical ideas and facts more easily if they see them demonstrated visually rather than having them stated in words. Describe a visual way to demonstrate the Angle Sum Theorem.

Sample answer: Cut off the angles of a triangle and place them side-by-side on one side of a line so that their vertices meet at a common point. The result will show three angles whose measures add up to 180.

**Finding Angle Measures in Triangles**

You can use algebra to solve problems involving triangles.

**Example** In triangle $ABC$, $m\angle A$ is twice $m\angle B$, and $m\angle C$ is 8 more than $m\angle B$. What is the measure of each angle?

Write and solve an equation. Let $x = m\angle B$.

$m\angle A + m\angle B + m\angle C = 180$

$2x + x + (x + 8) = 180$

$4x + 8 = 180$

$4x = 172$

$x = 43$

So, $m\angle A = 86$, $m\angle B = 43$, and $m\angle C = 43 + 8 = 51$.

**Solve each problem.**

1. In triangle $DEF$, $m\angle E$ is three times $m\angle D$, and $m\angle F$ is 9 less than $m\angle E$. What is the measure of each angle?

$m\angle D = 27$, $m\angle E = 81$, $m\angle F = 72$

2. In triangle $RST$, $m\angle T$ is 5 more than $m\angle R$, and $m\angle S$ is 10 less than $m\angle T$. What is the measure of each angle?

$m\angle R = 60$, $m\angle S = 55$, $m\angle T = 65$

3. In triangle $JKL$, $m\angle K$ is four times $m\angle J$, and $m\angle L$ is five times $m\angle J$. What is the measure of each angle?

$m\angle J = 18$, $m\angle K = 72$, $m\angle L = 90$

4. In triangle $XYZ$, $m\angle Z$ is 2 more than twice $m\angle X$, and $m\angle Y$ is 7 less than twice $m\angle X$. What is the measure of each angle?

$m\angle X = 37$, $m\angle Y = 67$, $m\angle Z = 76$

5. In triangle $GHI$, $m\angle H$ is 20 more than $m\angle G$, and $m\angle G$ is 8 more than $m\angle I$. What is the measure of each angle?

$m\angle G = 56$, $m\angle H = 76$, $m\angle I = 48$

6. In triangle $MNO$, $m\angle M$ is equal to $m\angle N$, and $m\angle M$ is 5 more than three times $m\angle N$. What is the measure of each angle?

$m\angle M = m\angle N = 35$, $m\angle O = 110$

7. In triangle $STU$, $m\angle U$ is half $m\angle T$, and $m\angle S$ is 30 more than $m\angle T$. What is the measure of each angle?

$m\angle S = 90$, $m\angle T = 60$, $m\angle U = 30$

8. In triangle $PQR$, $m\angle P$ is equal to $m\angle Q$, and $m\angle R$ is 24 less than $m\angle P$. What is the measure of each angle?

$m\angle P = m\angle Q = 68$, $m\angle R = 44$

9. Write your own problems about measures of triangles.

See students’ work.
4-3 Study Guide and Intervention

**Congruent Triangles**

**Identify Congruence Transformations**
Two triangles are congruent if and only if all three pairs of corresponding angles are congruent and all three pairs of corresponding sides are congruent. In the figure, \( \triangle ABC = \triangle RST \).

**Example**
If \( \triangle XYZ \equiv \triangle RST \), name the pairs of congruent angles and congruent sides.
\[
\begin{align*}
\angle X &= \angle R, \quad \angle Y &= \angle S, \quad \angle Z &= \angle T \\
XY &= RS, \quad XZ &= RT, \quad YZ &= ST
\end{align*}
\]

**Exercises**

Identify the congruent triangles in each figure.

1. \( \triangle ABC = \triangle JKL \)
2. \( \triangle ABC = \triangle DCB \)
3. \( \triangle JKM = \triangle LMK \)

Name the corresponding congruent angles and sides for the congruent triangles.

4. \( \angle E = \angle J; \quad \angle F = \angle K; \quad \angle G = \angle L; \quad EF = JK; \quad EG = JL; \quad FG = KL \)
5. \( \angle A = \angle D; \quad \angle ABC = \angle DCB; \quad \angle ACB = \angle BDC; \quad \angle ABC = \angle DCB; \quad \angle ACB = \angle BDC; \quad AB = DC; \quad AC = DB; \quad BC = CB \)
6. \( \angle R = \angle T; \quad \angle RSU = \angle TSU; \quad \angle RUS = \angle TUS; \quad RS = TU; \quad SU = TS \)

**Lessons**

Answers (Lesson 4-3)
Identify the congruent triangles in each figure.

1. \( \triangle JPL = \triangle TVS \)

2. \( \triangle ABC = \triangle WXY \)

3. \( \triangle PQR = \triangle PSR \)

4. \( \triangle DEF = \triangle DGF \)

Name the congruent angles and sides for each pair of congruent triangles.

5. \( \triangle ABC = \triangle FGH \)
   - \( \angle A = \angle F, \angle B = \angle G, \angle C = \angle H; AB = FG, BC = GH, AC = FH \)

6. \( \triangle PQR = \triangle STU \)
   - \( \angle P = \angle S, \angle Q = \angle T, \angle R = \angle U; PQ = ST, QR = TU, PR = SU \)

Verify that each of the following transformations preserves congruence, and name the congruence transformation.

7. \( \triangle ABC = \triangle A'BC' \)
   - \( AB = 2\sqrt{2}, A'B' = 2\sqrt{2}, \)
   - \( BC = 2\sqrt{2}, B'C' = 2\sqrt{2}, \)
   - \( AC = 4, A'C' = 4, \angle A = \angle A', \)
   - \( \angle B = \angle B', \angle C = \angle C'; \) slide

8. \( \triangle DEF = \triangle D'E'F' \)
   - \( DE = 4, D'E' = 4, EF = 5, \)
   - \( E'F' = 5, DF = 3, D'F' = 3, \)
   - \( \angle D = \angle D', \angle E = \angle E', \)
   - \( \angle F = \angle F'; \) flip

8. Indicate the triangles that appear to be congruent.
   \( \triangle ABI = \triangle EBF, \triangle CBD = \triangle HBG \)

8. Name the congruent angles and congruent sides of a pair of congruent triangles.
   Sample answer: \( \angle A = \angle E, \angle ABI = \angle EBF, \angle I = \angle F, \)
   \( AB = EB, BI = BF, AI = EF \)
4-3 Reading to Learn Mathematics

Congruent Triangles

Pre-Activity Why are triangles used in bridges?

Read the introduction to Lesson 4-3 at the top of page 192 in your textbook.

In the bridge shown in the photograph in your textbook, diagonal braces were used to divide squares into two isosceles right triangles. Why do you think these braces are used on the bridge? Sample answer: The diagonal braces make the structure stronger and prevent it from being deformed when it has to withstand a heavy load.

Reading the Lesson

1. If \( \triangle RST \cong \triangle UVW \), complete each pair of congruent parts.
   \[
   \angle R = \angle U \\
   \angle S = \angle W \\
   \angle T = \angle V \\
   RS = UW \\
   ST = WV
   \]

2. Identify the congruent triangles in each diagram.
   a. \( \triangle ABC \cong \triangle ADC \)
   b. \( \triangle PQR \cong \triangle QRS \)
   c. \( \triangle MNO \cong \triangle QPO \)
   d. \( \triangle RTV \cong \triangle USV \)

3. Determine whether each statement says that congruence of triangles is reflexive, symmetric, or transitive.
   a. If the first of two triangles is congruent to the second triangle, then the second triangle is congruent to the first. symmetric
   b. If there are three triangles for which the first is congruent to the second and the second is congruent to the third, then the first triangle is congruent to the third. transitive
   c. Every triangle is congruent to itself. reflexive

Helping You Remember

4. A good way to remember something is to explain it to someone else. Your classmate Ben is having trouble writing congruence statements for triangles because he thinks he has to match up three pairs of sides and three pairs of angles. How can you help him understand how to write correct congruence statements more easily? Sample answer: Write the three vertices of one triangle in any order. Then write the corresponding vertices of the second triangle in the same order. If the angles are written in the correct correspondence, the sides will automatically be in the correct correspondence also.
SSS Postulate You know that two triangles are congruent if corresponding sides are congruent and corresponding angles are congruent. The Side-Side-Side (SSS) Postulate lets you show that two triangles are congruent if you know only that the sides of one triangle are congruent to the sides of the second triangle.

If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.

Example

For each diagram, determine which pairs of triangles can be proved congruent by the SAS Postulate.

Example

1. Given
2. Given
3. Definition of midpoint
4. Reflexive Property of =
5. SSS Postulate

Write a two-column proof.

Given: \( \overline{AB} = \overline{DB} \) and \( C \) is the midpoint of \( \overline{AB} \).
Prove: \( \triangle ABC = \triangle DBC \)

Exercises

Write a two-column proof.

Given: \( \overline{AB} = \overline{XY} \), \( \overline{AC} = \overline{XZ} \), \( \overline{BC} = \overline{YZ} \).
Prove: \( \triangle ABC = \triangle XYZ \)

Exercises

For each figure, determine which pairs of triangles can be proved congruent by the SAS Postulate.

1. \( \triangle TRU = \triangle PMN \) by the SAS Postulate.
2. \( \angle XQY \) and \( \angle WQZ \) are not the included angles for the congruent segments. The triangles are not congruent by the SAS Postulate.
3. \( \angle MPL = \angle NPL \) because both are right angles. \( \triangle MPL \) and \( \triangle NPL \) are congruent by the SAS Postulate.
4. The triangles cannot be proved congruent by the SAS Postulate.
5. \( \angle D = \angle B \) because both are right angles. The two triangles are congruent by the SAS Postulate.
6. The congruent angles are the included angles for the congruent sides. \( \triangle FJH \) and \( \triangle GHJ \) are congruent by the SAS Postulate.
**Skills Practice**

**Proving Congruence—SSS, SAS**

Determine whether \( \triangle ABC \cong \triangle KLM \) given the coordinates of the vertices. Explain.

1. \( A(-3, 3), B(-1, 3), C(1, 1), K(1, 4), L(3, 4), M(1, 6) \)
   
   \( AB = 2, KL = 2, BC = 2\sqrt{2}, LM = 2\sqrt{2}, AC = 2, KM = 2. \) 
   
   The corresponding sides have the same measure and are congruent, so \( \triangle ABC \cong \triangle KLM \) by SSS.

2. \( A(-4, -2), B(-4, 1), C(-1, -1), K(0, -2), L(0, 1), M(4, 1) \)
   
   \( AB = 3, KL = 3, BC = \sqrt{13}, LM = 4, AC = \sqrt{10}, KM = 5. \) 
   
   The corresponding sides are not congruent, so \( \triangle ABC \) is not congruent to \( \triangle KLM \).

3. Write a flow proof.
   
   **Given:** \( PR = DE, PT = DF \)
   
   **Prove:** \( \triangle PRT \cong \triangle DEF \)

   **Proof:**
   
   \[
   \begin{align*}
   PR &= DE \quad \text{Given} \\
   PT &= DF \quad \text{Given} \\
   \angle R &= \angle E \quad \text{Reflexive Property} \\
   \angle T &= \angle F \quad \text{Definition of midpoint} \\
   \triangle PRT &= \triangle DEF \quad \text{SSS}
   \end{align*}
   \]

4. Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove that they are congruent, write **not possible**.

   4. SSS
   5. SAS
   6. not possible

**Answers**

4. Since \( \angle A'CB' \) and \( \angle ABC \) are vertical angles, they are congruent. In the figure, \( \angle A' \cong \angle A' \) and \( \angle B' \cong \angle B \). So \( \triangle AB'C \cong \triangle A'B'C \) by SAS. By CPCTC, the lengths \( A'B' \) and \( AB \) are equal.
Pre-Activity How do land surveyors use congruent triangles? Read the introduction to Lesson 4-4 at the top of page 200 in your textbook. Why do you think that land surveyors would use congruent right triangles rather than other congruent triangles to establish property boundaries? Sample answer: Land is usually divided into rectangular lots, so their boundaries meet at right angles.

Reading the Lesson
1. Refer to the figure.
   a. Name the sides of \( \triangle LMN \) for which \( \angle L \) is the included angle. \( LM, LN \)
   b. Name the sides of \( \triangle LMN \) for which \( \angle N \) is the included angle. \( NL, NM \)
   c. Name the sides of \( \triangle LMN \) for which \( \angle M \) is the included angle. \( ML, MN \)

2. Determine whether you have enough information to prove that the two triangles in each figure are congruent. If so, write a congruence statement and name the congruence postulate that you would use. If not, write \textit{not possible}.
   a. \( \triangle ABD \cong \triangle CBD; \text{SAS} \)
   b. not possible
   c. \( EH \) and \( DG \) bisect each other.
   d. \( \triangle DEF \cong \triangle GHF; \text{SAS} \)
   \( \triangle RSU \cong \triangle TSU; \text{SSS} \)

Helping You Remember
3. Find three words that explain what it means to say that two triangles are congruent and that can help you recall the meaning of the SSS Postulate. Sample answer: Congruent triangles are triangles that are the \textit{same size and shape}, and the SSS Postulate ensures that two triangles with three corresponding sides congruent will be the same size and shape.

4-4 Enrichment

**Congruent Parts of Regular Polygonal Regions**
Congruent figures are figures that have exactly the same size and shape. There are many ways to divide regular polygonal regions into congruent parts. Three ways to divide an equilateral triangular region are shown. You can verify that the parts are congruent by tracing one part, then rotating, sliding, or reflecting that part on top of the other parts.

1. Divide each square into four congruent parts. Use three different ways. \textit{Sample answers are shown.}

2. Divide each pentagon into five congruent parts. Use three different ways. \textit{Sample answers are shown.}

3. Divide each hexagon into six congruent parts. Use three different ways. \textit{Sample answers are shown.}

4. What hints might you give another student who is trying to divide figures like those into congruent parts? \textit{See students’ work.}
ASA Postulate

The Angle-Side-Angle (ASA) Postulate lets you show that two triangles are congruent.

**Example**

Find the missing congruent parts so that the triangles can be proved congruent by the ASA Postulate. Then write the triangle congruence.

a. 

Two pairs of corresponding angles are congruent, \( \angle A = \angle D \) and \( \angle C = \angle F \). If the included sides \( AC \) and \( DF \) are congruent, then \( \triangle ABC \cong \triangle DEF \) by the ASA Postulate.

b. 

\( \angle R = \angle Y \) and \( \overline{SR} = \overline{XY} \). If \( \angle S = \angle X \), then \( \triangle RST \cong \triangle YXW \) by the ASA Postulate.

**Exercises**

What corresponding parts must be congruent in order to prove that the triangles are congruent by the ASA Postulate? Write the triangle congruence statement.

1. \( \overline{DC} \cong \overline{BC}; \ \angle DCE \cong \angle BCA \)
2. \( \overline{WY} \cong \overline{WY}; \ \angle XYW \cong \angle ZYW; \ \angle WXY = \angle WZY \)
3. \( \angle ABE \cong \angle CBD; \ \overline{ABE} \cong \overline{CBD} \)
4. \( \overline{BD} \cong \overline{DB}; \ \angle ADB \cong \angle CBD; \ \angle ABD \cong \angle CDB \)
5. \( \overline{ST} \cong \overline{VT}; \ \angle RST \cong \angle UVT \)
6. \( \angle ACB \cong \angle E; \ \angle ABC \cong \angle CDE \)

**AAS Theorem**

Another way to show that two triangles are congruent is the Angle-Side-Angle (AAS) Theorem.

**Example**

In the diagram, \( \angle BCA = \angle DCA \). Which sides are congruent? Which additional pair of corresponding parts needs to be congruent for the triangles to be congruent by the AAS Postulate?

\( AC \cong AC \) by the Reflexive Property of Congruence. The congruent angles cannot be \( \angle 1 \) and \( \angle 2 \), because \( \angle C \) would be the included side. If \( \angle B = \angle D \), then \( \triangle ABC \cong \triangle ADC \) by the AAS Theorem.

**Exercises**

In Exercises 1 and 2, draw and label \( \triangle ABC \) and \( \triangle DEF \). Indicate which additional pair of corresponding parts needs to be congruent for the triangles to be congruent by the AAS Theorem.

1. \( \angle A = \angle D; \ \angle B = \angle E \)
2. \( \overline{BC} = \overline{EF}; \ \angle A = \angle D \)

If \( \overline{BC} \cong \overline{EF} \) (or if \( \angle AC \cong \angle DF \)), then \( \triangle ABC \cong \triangle DEF \) by the AAS Theorem.

If \( \angle C = \angle F \) (or if \( \overline{AB} \cong \overline{EF} \)), then \( \triangle ABC \cong \triangle DEF \) by the AAS Theorem.

3. Write a flow proof.

   **Given:** \( \angle S \cong \angle U; \ TR \) bisects \( \angle STU \).

   **Prove:** \( \overrightarrow{SRT} \cong \overrightarrow{URT} \)

   **Proof:**

   - \( TR \) bisects \( \angle STU \) (given)
   - \( \angle SRT \cong \angle URT \) (definition of bisector)
   - \( \angle SRT \cong \angle URT \) (by AAS Theorem)
   - \( \overrightarrow{SRT} \cong \overrightarrow{URT} \) (CPCTC)
Proving Congruence—ASA, AAS

1. Given:

   \( \angle L = \angle K \)
   \( \angle J = \angle N \)
   \( \angle S = \angle Q \)

   Prove: \( \triangle JLS \cong \triangle KNQ \)

   Sample proof:

   \( \angle TSU \)

   So \( \triangle TSU \cong \triangle TSU \) by \( \text{Reflexive Property} \).

   Therefore \( \triangle JLS \cong \triangle KNQ \) by \( \text{ASA} \).

2. Write a paragraph proof.

   Given: \( \angle D = \angle J \)
   \( \angle E = \angle F \)
   \( \angle G = \angle H \)

   Prove: \( \triangle DEF \cong \triangle GEF \)

   Proof: Since it is given that \( \angle D = \angle J \), \( \angle E = \angle F \), and \( \angle G = \angle H \), it is given that \( \angle D = \angle J \), \( \angle E = \angle F \), and \( \angle G = \angle H \). By the Reflexive Property, \( \triangle DFE \cong \angle DFE \) by \( \text{ASA} \).

   Therefore \( \triangle DEF \cong \triangle GEF \) by \( \text{ASA} \).

3. Write a paragraph proof.

   Given: \( AB = CD \)
   \( AD = AB \)
   \( \angle DAF = \angle CDB \)

   Prove: \( \triangle ADF \cong \triangle CDB \)

   Proof: Since it is given that \( AD = AB \) and \( \angle DAF = \angle CDB \), the \( \text{angle \ angle \ side} \) \( \text{property} \) tells us that \( \triangle ADF \cong \triangle CDB \).

4. Determine whether \( \triangle DEF \cong \triangle GEF \).

   Given: \( \angle D = \angle J \)
   \( \angle E = \angle F \)
   \( \angle G = \angle H \)

   Proof: Since it is given that \( \angle D = \angle J \), \( \angle E = \angle F \), and \( \angle G = \angle H \), it is given that \( \angle D = \angle J \), \( \angle E = \angle F \), and \( \angle G = \angle H \). By the Reflexive Property, \( \triangle DFE \cong \triangle DFE \) by \( \text{ASA} \).

   Therefore \( \triangle DEF \cong \triangle GEF \) by \( \text{ASA} \).
Reading to Learn Mathematics

Proving Congruence—ASA, AAS

Pre-Activity  How are congruent triangles used in construction?

Read the introduction to Lesson 4-5 at the top of page 207 in your textbook. Which of the triangles in the photograph in your textbook appear to be congruent? Sample answer: The four right triangles are congruent to each other. The two obtuse isosceles triangles are congruent to each other.

Reading the Lesson

1. Explain in your own words the difference between how the ASA Postulate and the AAS Theorem are used to prove that two triangles are congruent.
   Sample answer: In ASA, you use two pairs of congruent angles and the included congruent sides. In AAS, you use two pairs of congruent angles and a pair of nonincluded congruent sides.

2. Which of the following conditions are sufficient to prove that two triangles are congruent?
   A. Two sides of one triangle are congruent to two sides of the other triangle.
   B. The three sides of one triangle are congruent to the three sides of the other triangle.
   C. The three angles of one triangle are congruent to the three angles of the other triangle.
   D. All six corresponding parts of two triangles are congruent.
   E. Two angles and the included side of one triangle are congruent to two sides and the included angle of the other triangle.
   F. Two sides and a nonincluded angle of one triangle are congruent to two sides and a nonincluded angle of the other triangle.
   G. Two angles and a nonincluded side of one triangle are congruent to two angles and the corresponding nonincluded side of the other triangle.
   H. Two sides and the included angle of one triangle are congruent to two sides and the included angle of the other triangle.
   I. Two angles and a nonincluded side of one triangle are congruent to two angles and a nonincluded side of the other triangle.

3. Determine whether you have enough information to prove that the two triangles in each figure are congruent. If so, write a congruence statement and name the congruence postulate or theorem that you would use. If not, write not possible.
   a. \( \triangle AEB \cong \triangle DEC; \text{ AAS} \)
   b. \( T \) is the midpoint of \( \overline{RU} \).
   \( \triangle RST \cong \triangle UVT; \text{ ASA} \)

Helping You Remember

4. A good way to remember mathematical ideas is to summarize them in a general statement. If you want to prove triangles congruent by using three pairs of corresponding parts, what is a good way to remember which combinations of parts will work? Sample answer: At least one pair of corresponding parts must be sides. If you use two pairs of sides and one pair of angles, the angles must be the included angles. If you use two pairs of angles and one pair of sides, then the sides must both be included by the angles or must both be corresponding nonincluded sides.
Isosceles Triangles

Properties of Isosceles Triangles
An isosceles triangle has two congruent sides. The angle formed by these sides is called the vertex angle. The other two angles are called base angles. You can prove a theorem and its converse about isosceles triangles.

- If two sides of a triangle are congruent, then the angles opposite those sides are congruent. (Isosceles Triangle Theorem)
- If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

Example 1
Find $x$.

$BC = BA$, so
$m \angle A = m \angle C$.

Isos. Triangle Theorem

5$ = 10 - 5$

Subtract 4 from each side.

$x = 15$

Add 10 to each side.

Example 2
Find $x$.

$m \angle S = m \angle T$, so

Isos. Triangle Theorem

3$x - 13 = 2x$

Subtract 12$x$ from each side.

$x = 13$

Add 13 to each side.

Exercises

Find $x$.

1. 35

2. 12

3. 15

4. 3

5. 20

6. 36

7. Write a two-column proof.

Given: $\angle 1 \cong \angle 2$

Prove: $AB = CB$

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\angle 1 \cong \angle 2$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $\angle 2 \cong \angle 3$</td>
<td>2. Vertical angles are congruent.</td>
</tr>
<tr>
<td>3. $\angle 1 \cong \angle 3$</td>
<td>3. Transitive Property of $\cong$</td>
</tr>
<tr>
<td>4. $AB = CB$</td>
<td>4. If two angles of a triangle are $\cong$, then the sides opposite the angles are $\cong$.</td>
</tr>
</tbody>
</table>

Properties of Equilateral Triangles
An equilateral triangle has three congruent sides. The Isosceles Triangle Theorem can be used to prove two properties of equilateral triangles.

1. A triangle is equilateral if and only if it is equiangular.
2. Each angle of an equilateral $\triangle$ measures 60°.

Example
Prove that if a line is parallel to one side of an equilateral triangle, then it forms another equilateral triangle.

Given: $PQ \parallel BC$.

Prove: $\triangle APQ$ is equilateral.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\triangle ABC$ is equilateral; $PQ \parallel BC$.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $m \angle A = m \angle B = m \angle C = 60$</td>
<td>2. Each $\angle$ of an equilateral $\triangle$ measures 60°.</td>
</tr>
<tr>
<td>3. $\angle 1 \cong \angle B$, $\angle 2 \cong \angle C$</td>
<td>3. If $\parallel$ lines, then corres. $\angle$s are $\cong$</td>
</tr>
<tr>
<td>4. $m \angle 1 = 60$, $m \angle 2 = 60$</td>
<td>4. Substitution</td>
</tr>
<tr>
<td>5. $\triangle APQ$ is equilateral.</td>
<td>5. If $\angle$ is eqiangular, then it is equilateral.</td>
</tr>
</tbody>
</table>

Exercises

Find $x$.

1. 10

2. 5

3. 10

4. 12

5. 10

6. 15

7. Write a two-column proof.

Given: $\triangle ABC$ is equilateral; $\angle 1 \cong \angle 2$.

Prove: $\angle ADB \cong \angle CDB$

<table>
<thead>
<tr>
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<tr>
<td>1. $\triangle ABC$ is equilateral.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $AB = CB$, $\angle A = \angle C$</td>
<td>2. An equilateral $\triangle$ has $\cong$ sides and $\cong$ angles.</td>
</tr>
<tr>
<td>3. $\angle 1 \cong \angle 2$</td>
<td>3. Given</td>
</tr>
<tr>
<td>4. $\triangle ABD \cong \triangle CBD$</td>
<td>4. ASA Postulate</td>
</tr>
<tr>
<td>5. $\angle ADB = \angle CDB$</td>
<td>5. CPCTC</td>
</tr>
</tbody>
</table>
4-6 Skills Practice
Isosceles Triangles

Refer to the figure.

1. If $\overline{AC} \cong \overline{AD}$, name two congruent angles.
   $\angle ACD \cong \angle CDA$

2. If $\overline{BE} \cong \overline{BC}$, name two congruent angles.
   $\angle BEC \cong \angle BCE$

3. If $\angle ERA \cong \angle EAB$, name two congruent segments.
   $\overline{EB} \cong \overline{EA}$

4. If $\angle EDA \cong \angle CDE$, name two congruent segments.
   $\overline{CE} \cong \overline{CD}$

$\triangle ABD$ is isosceles, $\triangle CDF$ is equilateral, and $m\angle AFD = 150$.
Find each measure.

5. $m\angle CFD = 60$
6. $m\angle AFB = 55$
7. $m\angle ABF = 70$
8. $m\angle A = 55$

In the figure, $\overline{PL} \parallel \overline{RL}$ and $\overline{LR} \parallel \overline{RR}$.
9. If $m\angle LRP = 100$, find $m\angle BRL = 20$
10. If $m\angle LPR = 34$, find $m\angle B = 68$

11. Write a two-column proof.
Given: $\overline{CD} \parallel \overline{CG}$
    $\overline{DE} \parallel \overline{GF}$
Prove: $\overline{CE} \cong \overline{CF}$

Proof:

<table>
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<td>1. Given</td>
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<tr>
<td>2. $\angle D \cong \angle G$</td>
<td>2. If 2 sides of a $\triangle$ are $\cong$, then the $\triangle$ opposite those sides are $\cong$.</td>
</tr>
<tr>
<td>3. $\overline{DE} \parallel \overline{GF}$</td>
<td>3. Given</td>
</tr>
<tr>
<td>4. $\triangle CDE \cong \triangle CGF$</td>
<td>4. SAS</td>
</tr>
<tr>
<td>5. $\overline{CE} \parallel \overline{CF}$</td>
<td>5. CPCTC</td>
</tr>
</tbody>
</table>

Triangles $GHI$ and $HJM$ are isosceles, with $\overline{GH} \cong \overline{MI}$
and $\overline{HJ} \cong \overline{MJ}$. Triangle $KLM$ is equilateral, and $m\angle HKM = 50$.
Find each measure.

5. $m\angle KML = 60$
6. $m\angle HMG = 70$
7. $m\angle GHM = 40$
8. If $m\angle HJM = 145$, find $m\angle MHJ = 17.5$
9. If $m\angle G = 67$, find $m\angle GHM = 46$

10. Write a two-column proof.
Given: $\overline{DE} \parallel \overline{BC}$
    $\angle 1 \equiv \angle 2$
Prove: $\overline{AB} \cong \overline{AC}$

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\overline{DE} \parallel \overline{BC}$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $\angle 1 \equiv \angle 4$</td>
<td>2. Corr. $\triangle$ are $\cong$.</td>
</tr>
<tr>
<td>$\angle 2 \equiv \angle 3$</td>
<td>3. Given</td>
</tr>
<tr>
<td>3. $\angle 1 \equiv \angle 2$</td>
<td>4. Congruence of $\triangle$ is transitive.</td>
</tr>
<tr>
<td>4. $\angle 3 \equiv \angle 4$</td>
<td>5. If 2 $\triangle$ of a $\triangle$ are $\cong$, then the sides opposite those $\triangle$ are $\cong$.</td>
</tr>
<tr>
<td>5. $\overline{AB} \equiv \overline{AC}$</td>
<td></td>
</tr>
</tbody>
</table>

11. SPORTS A pennant for the sports teams at Lincoln High School is in the shape of an isosceles triangle. If the measure of the vertex angle is 18, find the measure of each base angle.
   $81, 81$
Reading to Learn Mathematics

Isosceles Triangles

Pre-Activity How are triangles used in art?

Read the introduction to Lesson 4-6 at the top of page 216 in your textbook.

• Why do you think that isosceles and equilateral triangles are used more often than scalene triangles in art? Sample answer: Their symmetry is pleasing to the eye.

• Why might isosceles right triangles be used in art? Sample answer: Two congruent isosceles right triangles can be placed together to form a square.

Reading the Lesson

1. Refer to the figure.
   a. What kind of triangle is \( \triangle QRS \)? isosceles
   b. Name the legs of \( \triangle QRS \). QS, RS
   c. Name the base of \( \triangle QRS \). QR
   d. Name the vertex angle of \( \triangle QRS \). \( \angle S \)
   e. Name the base angles of \( \triangle QRS \). \( \angle Q, \angle R \)

2. Determine whether each statement is always, sometimes, or never true.
   a. If a triangle has three congruent sides, then it has three congruent angles always
   b. If a triangle is isosceles, then it is equilateral sometimes
   c. If a right triangle is isosceles, then it is equilateral never
   d. The largest angle of an isosceles triangle is obtuse. sometimes
   e. If a right triangle has a 45° angle, then it is isosceles. always
   f. If an isosceles triangle has three acute angles, then it is equilateral. sometimes
   g. The vertex angle of an isosceles triangle is the largest angle of the triangle. sometimes

3. Give the measures of the three angles of each triangle.
   a. an equilateral triangle 60, 60, 60
   b. an isosceles right triangle 45, 45, 90
   c. an isosceles triangle in which the measure of the vertex angle is 70 70, 55, 55
   d. an isosceles triangle in which the measure of a base angle is 70 70, 40
   e. an isosceles triangle in which the measure of the vertex angle is twice the measure of one of the base angles 90, 45, 45

Helping You Remember

4. If a theorem and its converse are both true, you can often remember them most easily by combining them into an "if-and-only-if" statement. Write such a statement for the Isosceles Triangle Theorem and its converse. Sample answer: Two sides of a triangle are congruent if and only if the angles opposite those sides are congruent.
4-7 Study Guide and Intervention
Triangles and Coordinate Proof

Position and Label Triangles A coordinate proof uses points, distances, and slopes to prove geometric properties. The first step in writing a coordinate proof is to place a figure on the coordinate plane and label the vertices. Use the following guidelines:

1. Use the origin as a vertex or center of the figure.
2. Place at least one side of the polygon on an axis.
3. Keep the figure in the first quadrant if possible.
4. Use coordinates that make the computations as simple as possible.

Example Position an equilateral triangle on the coordinate plane so that its sides are \(a\) units long and one side is on the positive \(x\)-axis. Start with \(R(0, 0)\). If \(RT\) is \(a\), then another vertex is \(T(a, 0)\).

For vertex \(S\), the \(x\)-coordinate is \(\frac{a}{2}\). Use \(b\) for the \(y\)-coordinate, so the vertex is \(S\left(\frac{a}{2}, b\right)\).

Exercises Find the missing coordinates of each triangle.

1. \(C(p, q)\)
2. \(T(2a, 2a)\)
3. \(E(-2g, 0), F(0, b)\)

<table>
<thead>
<tr>
<th>Position and label each triangle on the coordinate plane.</th>
</tr>
</thead>
<tbody>
<tr>
<td>4. isosceles triangle (\triangle RST) with base (RS) 4(a) units long</td>
</tr>
<tr>
<td>5. isosceles right (\triangle DEF) with legs (e) units long</td>
</tr>
<tr>
<td>6. equilateral triangle (\triangle EQI) with vertex (Q(0, a)) and sides 2(b) units long</td>
</tr>
</tbody>
</table>

Sample answers are given.

Exercises Prove that the segments joining the midpoints of the sides of a right triangle form a right triangle.

Sample answer: Position and label right \(\triangle ABC\) with the coordinates \(A(0, 0), B(0, 2b),\) and \(C(2a, 0)\).

The midpoint \(P\) of \(BC\) is \(\left(\frac{0 + 2a}{2}, \frac{2b + 0}{2}\right) = (a, b)\).

The midpoint \(Q\) of \(AC\) is \(\left(\frac{0 + 2a}{2}, \frac{0 + 0}{2}\right) = (a, 0)\).

The midpoint \(R\) of \(AB\) is \(\left(\frac{0 + 0}{2}, \frac{0 + 2b}{2}\right) = (0, b)\).

The slope of \(\overline{RP}\) is \(\frac{b - b}{a - 0} = \frac{0}{a} = 0\), so the segment is horizontal.

The slope of \(\overline{PQ}\) is \(\frac{b - 0}{a - a} = \frac{b}{0}\) which is undefined, so the segment is vertical.

\(\angle RPQ\) is a right angle because any horizontal line is perpendicular to any vertical line. \(\triangle PRQ\) has a right angle, so \(\triangle PRQ\) is a right triangle.
4-7 Skills Practice
Triangles and Coordinate Proof

Find the missing coordinates of each triangle.

1. Right \( \triangle FGH \) with legs \( a \) units and \( b \) units
2. Isosceles \( \triangle KLP \) with base \( KP \) 66 units long
3. Isosceles \( \triangle AND \) with base \( AD \) 5a long
4. \( A(0, 2a) \)
5. \( Z(b, c) \)
6. \( M(0, c) \)
7. \( Q(4a, 0) \)
8. \( R\left(\frac{7}{2}b, c\right) \)
9. \( T(0, b) \)

10. Write a coordinate proof to prove that in an isosceles right triangle, the segment from the vertex of the right angle to the midpoint of the hypotenuse is perpendicular to the hypotenuse.

Given: Isosceles right \( \triangle ABC \) with \( \angle ABC \) the right angle and \( M \) the midpoint of \( \overline{AC} \)

Prove: \( \overline{BM} \perp \overline{AC} \)

Proof:

The Midpoint Formula shows that the coordinates of \( M \) are \( \left(\frac{0 + 2a}{2}, \frac{2a + 0}{2}\right) \) or \((a, a)\). The slope of \( \overline{AC} \) is \( \frac{2a - 0}{0 - 2a} = -\frac{1}{2} \). The slope of \( \overline{BM} \) is \( \frac{a - 0}{a - 0} = 1 \). The product of the slopes is \(-1\), so \( \overline{BM} \perp \overline{AC} \).

Sample answers are given.

NEIGHBORHOODS For Exercises 7 and 8, use the following information.
Karina lives 6 miles east and 4 miles north of her high school. After school she works part time at the mall in a music store. The mall is 2 miles west and 3 miles north of the school.

7. Write a coordinate proof to prove that Karina's high school, her home, and the mall are at the vertices of a right triangle.

Given: \( \triangle SKM \)

Prove: \( \triangle SKM \) is a right triangle.

Proof:

Slope of \( \overline{SK} = \frac{4 - 0}{6 - 0} = \frac{2}{3} \)

Slope of \( \overline{SM} = \frac{3 - 0}{-2 - 0} = -\frac{3}{2} \)

Since the slope of \( \overline{SM} \) is the negative reciprocal of the slope of \( \overline{SK} \), \( \overline{SM} \perp \overline{SK} \). Therefore, \( \triangle SKM \) is right triangle.

8. Find the distance between the mall and Karina’s home.

\[ KM = \sqrt{(-2 - 6)^2 + (3 - 4)^2} = \sqrt{64 + 1} = \sqrt{65} \approx 8.1 \text{ miles} \]
4-7 Reading to Learn Mathematics

Read the introduction to Lesson 4-7 at the top of page 222 in your textbook.

Pre-Activity How can the coordinate plane be useful in proofs?

From the coordinates of A, B, and C in the drawing in your textbook, what do you know about ΔABC? Sample answer: ΔABC is isosceles with ∠C as the vertex angle.

Reading the Lesson

1. Find the missing coordinates of each triangle.
   a. R(0, b), S(0, 0), T(\(a, \frac{b}{2}\))
   b. D(0, 0), E(0, a), F(a, a)

2. Refer to the figure.
   a. Find the slope of SR and the slope of ST. \(1; -1\)
   b. Find the product of the slopes of SR and ST. What does this tell you about SR and ST? \(-1; SR \perp ST\)
   c. What does your answer from part b tell you about ΔRST?
      Sample answer: ΔRST is a right triangle with \(\angle S\) as the right angle.
   d. Find SR and ST. What does this tell you about SR and ST?
      \(SR = \sqrt{2a^2} \text{ or } a\sqrt{2}; ST = \sqrt{2a^2} \text{ or } a\sqrt{2}; SR = ST\)
   e. What does your answer from part d tell you about ΔRST?
      Sample answer: ΔRST is isosceles with \(\angle RST\) as the vertex angle.
   f. Combine your answers from parts c and e to describe ΔRST as completely as possible.
      Sample answer: ΔRST is an isosceles right triangle. \(\angle RST\) is the right angle and is also the vertex angle.
   g. Find \(m\angle SRT\) and \(m\angle STR\). 45; 45
   h. Find \(m\angle OSR\) and \(m\angle OST\). 45; 45

Helping You Remember

3. Many students find it easier to remember mathematical formulas if they can put them into words in a compact way. How can you use this approach to remember the slope and midpoint formulas easily?
   Sample answer: Slope Formula: change in y over change in x;
   Midpoint Formula: average of x-coordinates, average of y-coordinates

4-7 Enrichment

How Many Triangles?

Each puzzle below contains many triangles. Count them carefully. Some triangles overlap other triangles.

How many triangles are there in each figure?

1. 8
2. 40
3. 35
4. 5
5. 13
6. 27

How many triangles can you form by joining points on each circle? List the vertices of each triangle.

7. 4; ABC, ABD, ACD, BCD
8. 10; EFG, EFH, EHI, EGH, EHI, FGI, FHI, EGI, GHI
9. 20; JKL, JKM, JKN, JKO, JLM, JLN, JLO, JMN, JMO, KLM, KLN, KLO, KMN, KMO, KNO, LMIN, LMO, LNO, MNO
10. 35; PQR, PQS, PQT, PQU, PQV, PRS, PRT, PRU, PRV, PST, PSU, PSV, PTV, PTV, PV, QRS, QRT, QRU, QRV, QST, QSU, QSV, QTU, QTV, QUV, RST, RSU, RSV, RTU, RTV, RUV, STU, STV, STU, SUV, TUV